

# Information Aggregation and Optimal Structure of the Executive\*

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## Abstract

This paper provides a novel model of executives in Parliamentary democracies. We account for key features of these institutions: decision-making authority is assigned to individual ministers; the parliamentary majority provides support for this assignment; and the parliament debates policy. Supposing that politicians' private information is relevant for all policies—the 'common state' case—we show that cabinet meetings Pareto dominate private conversations between policymakers. In large governments, we show that authority should be concentrated to the most moderate politicians. In numerical simulations describing smaller governments, we find that a single leader should be assigned a large share of decisions. Turning to the case in which politicians have policy specific expertise, surprisingly, we find that the optimal executive structure is no less centralized than in the common-state case. Our results recover stylized facts—ideologically diverse parliaments and centralized executive control—of parliamentary governance first analyzed with respect to political development in Victorian England.

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## 1. INTRODUCTION

The cornerstone of democratic legitimacy is the consent given to those who exercise decision-making authority. An empirical regularity in all representative polities, observed most clearly in its parliamentary form, is a centralization of executive authority that sits alongside a diversity of views held within the parliamentary body. For example, whilst the 19th century House of Commons was divided into parliamentary factions, during this period, legislative and executive powers were fused in a cabinet lead by a dominant Prime Minister. These stylized and enduring facts – centralized authority in internally divided parliaments – demand explanation. To provide one, we develop a formal model that builds on key features and functions of parliamentary democracy, that are most well developed in the Westminster system.

First, a critical feature of parliamentary democracy is that the responsibility for initiation and implementation of specific policies lies with individual ministers who are allocated a departmental brief. Second, Parliament must consent to this allocation of decision-making powers. Third, the Parliament plays an advisory role to the executive via debate and communication. We model these key elements to provide a novel account of parliamentary democracy that explores how the structure of the executive facilitates effective aggregation of information and decision-making. Whilst the allocation of decision-making rights is a core feature of models of government formation ([Austen-Smith and Banks, 1990](#); [Laver and Shepsle, 1990, 1996](#)), and the role that parliament plays in providing consent for the executive has previously been explored ([Cox, 1987](#); [Diermeier and Feddersen, 1998](#)), our work is the first that combines these features in a model of information aggregation that, in addition, accounts for the advisory role of the Parliament.

The bare-bones of our model are a set of ideologically-differentiated politicians (a parliamentary majority) and a set of policy decisions to be implemented (the government programme). The parliamentary majority faces a collective choice problem on the assignment of decision-making authority over items in its programme: each policy can be assigned to at most one politician, though a politician may exercise authority on more than one policy. Information relevant to policy is dispersed amongst the set of politicians and may be common to all policies or specific to individual policies. Conditional on the assignment of authority, each politician chooses whether to communicate her information to decision-makers before they implement their policies.

An equilibrium of our game consists of the communication structure of the majority party together with a set of policy outcomes. Our focus is on the equilibria that maximize the majority's welfare *ex-ante*. We then calculate the optimal assignment of authority: the one that reflects the diversity of viewpoints held within the party and that leads to communication within the parliament that aggregates the most information. Our analysis of the optimal executive form provides reference to several critical elements: the executive's size refers to the number of politicians granted decision-making authority; its composition distinguishes those who exercise authority from those who do not; whilst its balance refers to the number of policies assigned to different ministers.

An additional element of the executive structure, and an important primitive in our model, is the form taken by communication. Under private communication, an assembly member can separately convey her message to each decision maker; under public communication any such communication is publicly known to all members of the Executive. We propose that this conceptual distinction captures an important element in the difference between a *government* of ministers and a *cabinet* of ministers: both collective pronouns refer to a multimember executive body; the critical distinction is that the latter involves a regular meeting at a designated time and place. During such deliberations, policy relevant information held by one minister is made available to all who exercise executive authority. Adopting this distinction, we explore the effects of cabinet deliberation on strategic communication by politicians to members of the cabinet.

The first part of our analysis concerns the case in which all information is relevant for all policies (the 'common state' model). We show that in the absence of a cabinet, so that politicians can only communicate information to executive holders in private, the optimal assignment grants *all* decision-making authority to a unique individual. This result is based on the finding that the willingness of one player to truthfully communicate information to a minister does not depend on the assignment of authority to any other minister combined with the stipulation that every politicians' information is relevant for all policies. Our surprising "full centralization" result under private communication may, however, revert once we allow for public meetings. In that case it may prove optimal for decision-making authority to be shared between ministers. In fact, whilst a politician may be unwilling to communicate truthfully to a single leader who is ideologically distant, she may be truthful when power is shared with another cabinet member whose ideology is intermediate. This apparently innocuous observation leads to a powerful normative result: public

meetings, typical of cabinet governance, dominate private conversations with policy makers. We view this result as laying formal normative foundations for cabinet government.

These results prompt the question: to what extent should decision-making authority be centralized within the cabinet? First we calculate the optimal executive in the simple case of three players. Doing so we show that, although full centralization (to the central politician) is optimal in a large parameter space, there are regions where power is shared with one or even both extreme politicians. Next, exploring the limit as the number of politicians becomes large, we show that all decision making authority should be concentrated to politicians who are ideologically close to the most moderate one. Finally, numerical simulations—randomly drawing ideology profiles and calculating the optimal policy assignment—of an intermediately sized parliament show that even with public communication in a cabinet environment fully centralized authority is fairly frequent. Moreover, even when it is optimal for authority to be shared, the balance of power in a multi-member cabinet is highly uneven: a single minister (perhaps a Prime Minister) should be assigned a large share (on average at least 80 % of decisions). Combining these insights, the implication of our analysis is that, by and large, the normative underpinnings for centralized authority, established for the case of private conversations, continue to hold when cabinet deliberates over outcomes.

Having shown that an optimally designed executive involves centralized authority, we then ask what are the characteristics of those who wield power. We uncover two important forces behind the selection of executive leaders: the need for moderation on the one hand, and for effective aggregation of information on the other. The first is intuitive and, indeed, is a key implication of many models of collective choice. Put simply, decision-making authority should be assigned to those less extreme in their views, since policies they implement will reflect the wider views of the assembly. The second, more novel and less obvious, force highlights strategic incentives for politicians to share information that are stronger for those of similar ideology. Simply put, we find that an important element in granting decision-making authority to an individual is the number of ideologically close-minded allies she has.

In the second part of the analysis we move away from the assumption that all politicians' information is relevant to all policies. Indeed we consider the polar opposite case: each politician has a different expertise, and is therefore informed only about one particular policy. Does highly dispersed expertise lead to decentralization? Surprisingly not. We find that full decentralization is

*never* the optimal decision-making authority assignment. In fact, all policy decisions should be granted to the most moderate politician, unless the policy expert has intermediate ideology, (that is, he neither too moderate, nor too extreme). The rationale for this result is simple. If moderate policy experts are willing to communicate to a more moderate politician then it is optimal to reassign such policies (to the more moderate politician). If extreme policy experts are willing to communicate only with extreme politicians then the parliamentary majority is better off letting the (uninformed) most moderate politician decide. Only in the intermediate case is it optimal for the expert to decide. Characterizing the optimal assignment, we first show that the fraction of decisions assigned to the most moderate politician converges to unity in the case of large governments if the bias distribution becomes either very concentrated or very dispersed (under a mild condition on the bias distribution). Performing numerical simulations we find, surprisingly, that the optimal executive structure is no less centralized than in the common-state case.

In sum, our analysis shows that cabinet meetings outperform communication via private conversations, and that there is a strong tendency for concentration of authority in the optimal executive. These results recover key stylized facts of parliamentary democracy: the emergence of Cabinet government under a dominant Prime Minister despite ideological division in the parliamentary majority. These facts have been first documented and discussed for the case of Victorian England and the next section discusses the implications of our analysis in this context in more detail.

## 2. WHY CABINET GOVERNMENT?

The starting point of our analysis is the stylized fact that in parliamentary democracy the diverse preferences of an assembly sit alongside centralized decision-making authority. In the United Kingdom, the centralized executive has its origins in monarchical government. Parliamentary division was kept in check by the Prime Minister who effectively exercised patronage of the Crown and was the leader of a centralized executive. However, it was not always so. Authority was at times more widely dispersed. During Parliament’s “golden age” the power to initiate policy rested with individual members, and it was not until the late 19th century that decision-making authority was centralized in a cabinet that fused legislative and executive powers.

Cox (1987), building on Bagehot (1867), provides the classic account of this process. He relates centralized authority to distributional concerns owing to the extension of the franchise under the

Great Reform Acts of 1832 and 1867. These made MPs more responsive to constituents' concerns. The legislative process become less efficient as a result. Then centralization of the legislative initiative within a single party cabinet represented a Pareto improvement. In Cox's words:

“ Each MP wished to exercise the extraordinary parliamentary rights available to ventilate his or his constituents, grievances and opinions; but when too many did exercise their rights the cumulative effect was distressing to MPs. To extricate themselves from the dilemma in which they were entangled, the Commons repeatedly took the most obvious way out and abolished the rights that were being abused.”

We seek to understand the legitimacy of centralized authority, but do so from the perspective of information aggregation. We thus abstract from Parliament's role in distributing local public goods and services, and focus instead on the advise it provides to the executive.<sup>1</sup> This approach seems natural. Indeed the etymological origins of the word "parliament", a late 13th century word from the Old French *parlement*, the name of which is derived from *parler*- to speak, suggests a forum for the communication and exchange of information. [Bagehot \(1867\)](#), for example, wrote that the "modern" British Parliament that emerged in the nineteenth century maintained an "informative" function analogous to the role played by the "medieval" Parliament which advised the monarch. We explore the relationship between the informative function of the Parliament and the allocation of decision-making authority allowing for a wide range of cases including decentralized authority (akin to the golden age of parliament described above) and centralized authority either to a unique individual (a Prime Minister), or to a Cabinet or government of ministers.

Our account of centralized authority goes further. In particular we are able to address an important puzzle raised by Cox: why did centralization of authority take the particular form of cabinet government (Cox (1987), page 61)? Our answer highlights the deliberative aspects of cabinet governance. We view Cabinet as a physical entity – a meeting at a designated time and place – where executive members share information relevant to the policies that they will implement. Final policies depend on the exchange of information that takes place in cabinet meetings and this fact gives rise to strategic considerations at the heart of our model. We compare outcomes of a deliberative cabinet process to those in the absence of such a process. Although we do not

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<sup>1</sup>Further, our focus on single party government can be justified in order to abstract from the competitive party tensions that are important in Cox's work.

model ministers actions in a cabinet setting directly – we capture this cabinet process via a simple communication protocol whereby any information held by one cabinet minister is known to all – our reduced form view of the Cabinet as a meeting place where information is shared has a direct implication: no minister can abrogate himself from responsibility for government policy by claiming that he was unaware of the policy to be implemented, or the reasons behind it.<sup>2</sup> This implication is, we believe, central to the workings of collective responsibility that provides the defining feature of cabinet government.<sup>3</sup> Under collective responsibility, government ministers must support government policy or resign. As Cox (1994) shows that the convention by which anything a minister proposed to parliament was government policy, and thus has the cabinet seal of approval, was established practice by the time of Peel’s cabinet in 1841. A necessary condition for this convention to operate is that ministers understand what the government policy is (as defined by the minister responsible) and the reasons why that policy is pursued. The existence of a Cabinet allows this to be so. Put simply, the Cabinet is the place that enables ministers to informally develop the collective responsibility of the government that is required by convention.<sup>4</sup>

Our information aggregation perspective can also be used to evaluate other claims concerning the prevalence of cabinet decision-making in parliamentary democracies. Cox (1987) explains cabinet governance in Victorian England from historical precedent: from its inception as the Privy Council, of which it remains a part today, the Cabinet was the locus of existing government expertise. Whilst we believe that the argument for delegation on account of asymmetric expertise is common, it is not clear that such delegation is optimal in a Parliament. A key advantage of our setup is that we can adjust the primitives of our model- in particular whether uncertainty is common to all policies or specific to each policy- to analyze the effects of different degrees of dispersion of expertise. In a specification of our model we suppose that expertise is widely distributed amongst assembly members, so that each member is an expert on a particular policy. Foreshadowing our results, we are

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<sup>2</sup>In practice this is so *even* if the minister was not present in the cabinet meeting at which the policy decision was discussed, see the Cabinet Manual Draft (2010),p 54, Cabinet Office.

<sup>3</sup>It would be untrue to say that there exists no notion of collective responsibility in practice in congressional systems. Fiorina (1980), for example offers the perspective that American parties exercise limited collective responsibility. It does not exist in a constitutional sense as in the United Kingdom and other parliamentary democracies.

<sup>4</sup>The Privy Council Office of the Government of Canada draws a related distinction between a Ministry and a Cabinet: “The ministry is a term applied to ministers holding office at the pleasure of the Crown, and individually responsible in law to the Crown and by convention to the House of Commons for their activities. The cabinet is a place provided by the prime minister to enable his colleagues informally to develop the collective responsibility of the ministry required by the convention of the constitution. In a word, the cabinet is the prime minister’s cabinet and is the physical expression of collective responsibility. The ministry, on the other hand, summarizes the individual authority of its members.” see Responsibility in the Constitution, chapter 3, Minister of Supply and Services Canada 1993.

then able to show that when expertise is distributed across assembly members then decentralization of authority is never optimal and indeed we use simulations to show that outcomes are qualitatively similar to the case where expertise is more evenly distributed.

In sum, our model of information aggregation can breathe new life on a set of questions concerning the centralization of decision-making authority in parliamentary government: why does centralization occur? Why does it take the particular form of Cabinet government? And finally what are the characteristics of executive leaders? We briefly discuss the literature on which we build.

### 3. RELATED LITERATURE

This paper relates to a broader literature which that the effect of collective decision-making bodies on information aggregation. Most of this literature, building on the seminal contributions by [Austen-Smith and Banks \(1996\)](#) and [Feddersen and Pesendorfer \(1996, 1997, 1998\)](#), has focused on voting and information aggregation in committees. Our emphasis on a government in which policy decisions are implemented by individual ministers, (rather than voted on in committee), brings into sharp focus the optimal assignment of decision-making authority by a parliamentary majority. This focus is shared with other models of cabinet governance; in particular the portfolio-assignment models of [Laver and Shepsle \(1990, 1996\)](#) and [Austen-Smith and Banks \(1990\)](#). While those models are concerned with the spatial effects of different portfolio allocations with no uncertainty, we provide an information aggregation analysis that formalizes the advisory role of the Parliament. Our focus on the assignment of decision-making authority from an information aggregation perspective is related to [Dewan and Myatt \(2007a, 2008\)](#) who analyze the emergence of a centralized party leadership and the characteristics of those leaders in a common-value setting.

The model we propose is circumscribed by the cheap talk literature that builds on the seminal contribution of [Crawford and Sobel \(1982\)](#), applied in a political science setting by [Gilligan and Krehbiel \(1987\)](#) and extended by [Battaglini \(2002\)](#) amongst others. That literature has focussed primarily on congressional systems and on the stylized relationship between a unitary parent body (represented by the median floor member in the House) and a single committee which holds expertise. Our model of parliamentary democracy analyzes a richer situation in which multiple members of a Parliament communicate strategically with a government of ministers. Building on [Morgan and Stocken \(2008\)](#) and [Farrell and Gibbons \(1989\)](#), [Galeotti, Ghiglino, and Squintani \(2009\)](#) develop

a general theoretical framework to study multi-player communication and present applications in the economics of networks. The current paper expands that framework to study the specific issue of optimal government in parliamentary democracies. In particular this requires enriching the framework to allow for authority to be transferred across players and for agents to have specific information about some decisions but not others.

Our analysis provides insight into a tradeoff between moderation and information in the optimal allocation of authority. Moderation is also relevant in the single-sender world of Gilligan and Krehbiel. Their the adoption of restrictive procedural rules, that do not allow the parent body to amend legislation, can provide incentives for costly information acquisition. Such rules are optimal when experts' ideal policies are not too distant from the floor median. Our focus on ideological divergence in a parliament gives rise to an effect that balances the moderation requirement: strategic communication between an MP and a minister depends not only upon the ideological distance between them but also upon how many others communicate (truthfully) with the minister.

Our model contributes to a small but growing formal literature on executive leadership in parliamentary democracies. For the most part this literature has focussed on issues of moral hazard (Dewan and Myatt, 2007b, 2010; Indridason and Kam, 2008), though recent models of parliamentary democracy from an adverse selection perspective are by Huber and Martinez-Gallardo (2008) and Dewan and Hortalla-Valve (2011). The latter analyze strategic communication between ministers and a Prime Minister who makes appointments, allocates portfolios, and assigns different tasks to each portfolio; they also provide normative justification for centralized authority.

#### 4. MODEL

We consider the following information aggregation and collective decision problem. Suppose that a set of  $\mathcal{I} = \{1, \dots, I\}$  of politicians form a single-party Parliamentary majority. Their role is to provide consent for its governing executive. The majority is faced with the collective task of choosing an assignment  $a : \mathcal{K} \rightarrow \mathcal{I}$  of policy decisions. This assignment grants decision-making authority over a set of policies  $\mathcal{K}$ . For each  $k \in \mathcal{K} = \{1, \dots, K\}$ , the decision  $y_k$  is a policy on the left-right spectrum  $\mathfrak{R}$ . For simplicity we think of the assignment as granting complete jurisdiction over policy  $k$ , though of course other interpretations, such as, for example, the assignment of agenda-setting

rights could also be incorporated. The important element is that decision-making authority over each policy is granted by the collective body of politicians to a unique individual.

In a fully-decentralized executive each policy decision is assigned to a different politician so that  $a(k) \neq a(k')$  for all  $k, k'$  in  $\mathcal{K}$ . At the opposite end of the spectrum all decisions are centralized to a single leader so that  $a(k) = a(k')$  for all  $k, k'$  in  $\mathcal{K}$ . We let the range of  $a$  be denoted by  $a(\mathcal{K}) \subseteq \mathcal{I}$ , which we term as the set of politicians with decision-making authority. We sometimes refer to such politicians collectively as *active*, othertimes we refer to them individually as *ministers*. For any active politician  $j$ , we let  $a_j$  denote the number of policies she takes under assignment  $a$ . Our specification thus allows us to capture important elements of the executive body: its size—beyond the extremes of full decentralization and the leadership of one, there are a range of possibilities; and its balance—amongst the set of active politicians some may have more authority than others. Politicians are ideologically differentiated, and care about all policy choices made. For any policy decision  $\hat{y}_k$ , their preferences also depend on unknown states of the world  $\theta_k$ , uniformly distributed on  $[0, 1]$ . Specifically, were she to know the vector of states  $\theta = (\theta_k)_{k \in \mathcal{K}}$ , we specify politician  $i$ 's payoff as:

$$u_i(\hat{\mathbf{y}}, \theta) = - \sum_{k=1}^K (\hat{y}_k - \theta_k - b_i)^2.$$

Hence, each politician  $i$ 's ideal policy is  $\theta_k + b_i$ , where the bias  $b_i$  captures ideological differentiation, and we assume without loss of generality, that  $b_1 \leq b_2 \leq \dots \leq b_I$ . The vector of ideologies  $\mathbf{b} = \{b_1, \dots, b_I\}$  is common knowledge.

Each politician  $i$  has some private information on the vector  $\theta$ . Specifically, we make two opposite assumptions on politicians' information. Firstly, for some of our analysis we assume that uncertainty over all policies is captured by a single *common* state that represents the underlying economic and social fundamentals. For example, an underlying economic recession will influence policy choices of all ministries, from the Home office immigration policy, to the fiscal policy of the Chancellor of the Exchequer. We represent these fundamentals by a single uniformly distributed state of the world  $\theta$ , so that  $\theta_k = \theta$  for all  $k$ , and each politician  $i$ 's signal  $s_i$  is informative about  $\theta$ . Conditional on  $\theta$ ,  $s_i$  takes the value equal to one with probability  $\theta$  and to zero with probability  $1 - \theta$ . Secondly, and in an alternative specification we say that the politician's information is *policy specific*. Each policy has its own underlying set of circumstances over which politicians may be informed. Thus the random variables  $\theta_k$  are identical and independently distributed across  $k \in \mathcal{K}$ , and each politician

$k$  receives a signal  $s_k \in \{0, 1\}$  about  $\theta_k$  only, with  $\Pr(s_k = 1|\theta_k) = \theta_k$ . In the case of policy specific information, for simplicity, we take  $\mathcal{K}=\mathcal{I}$  so that each politician is informed on a single issue. This specification allows us to explore a situation where expertise on policies varies and is widely dispersed amongst the set of politicians.

In our set-up, the Parliament acts as a forum via which information can be aggregated and transmitted to policy makers. In order to aggregate information, politicians may communicate their signals to each other before policies are executed. We allow for communication to either take the form of private conversations, or public meetings. We might think of private communication as taking place over dinner, or via a secure communication network, with no leakage of information transmitted. Hence, each politician  $i$  may send a different message  $\hat{m}_{ij} \in \{0, 1\}$  to any politician  $j$ . Under public communication, by contrast, a politician is unable to communicate privately with a decision-maker as all communication is publicly available to those who exercise authority. Hence, each politician  $i$  sends the same message  $\hat{m}_i$  to all decision makers. A pure communication strategy of player  $i$  is a function  $m_i(s_i)$ .

As already noted, the distinction we draw between these different modes of communication captures a subtle but key difference in the type of executive body that forms. The assumption that under public communication any information made available to one minister is made available to all members of the executive captures the process of cabinet deliberations. As explained earlier, this forms an important element of the collective responsibility of the Cabinet.<sup>5</sup> Note however, that under our notion of cabinet government, decisions are still taken by individual ministers who have discretion up to the point where they make all information available. Ministers are not bound by a collective decision-making rule when implementing policy.

Communication between politicians allows information to be transferred. Up to relabeling of messages, each communication strategy from  $i$  to  $j$  may be either *truthful*, in that a politician reveals her signal to  $j$ , so that  $m_{ij}(s_i) = s_i$  for  $s_i \in \{0, 1\}$ , or “*babbling*”, and in this case  $m_{ij}(s_i)$  does not depend on  $s_i$ . Hence, the communication strategy profile  $\mathbf{m}$  defines the truthful communication network  $\mathbf{c}(\mathbf{m})$  according to the rule:  $c_{ij}(\mathbf{m}) = 1$  if and only if  $m_{ij}(s_i) = s_i$  for every  $s_i \in \{0, 1\}$ . This definition provides us with the communication structure of the party.

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<sup>5</sup>For example, successive enquiries into the second Gulf War, over which several senior ministers resigned rather than accept the collective responsibility of cabinet, raised concerns over whether the Prime Minister knowingly issued false information to his cabinet; indicating that if this were in fact the case, then this is an exception to the rule.

The second strategic element of our model involves the final policies implemented. Conditional on her information, the assigned decision-maker implements her preferred policy. We denote a policy strategy by  $i$  as  $y_k : \{0, 1\}^{\mathcal{I}} \rightarrow \Re$  for all  $k = a^{-1}(i)$ . Given the received messages  $\hat{\mathbf{m}}_{-i,i}$ , by sequential rationality, politician  $i$  chooses  $\hat{y}_k$  to maximize expected utility, for all  $k$  such that  $i = a(k)$ . So,

$$(1) \quad y_k(s_i, \hat{\mathbf{m}}_{i,-i}) = b_i + E[\theta_k | s_i, \hat{\mathbf{m}}_{-i,i}],$$

and this is due to the quadratic loss specification of players payoffs.

Given the assignment  $a$  an equilibrium then consists of  $(\mathbf{m}, \mathbf{y})$  and a set of beliefs that are consistent with equilibrium play. We use the further restriction that an equilibrium must be consistent with some beliefs held by politicians off the equilibrium path of play. Thus our equilibrium concept is pure-strategy Perfect Bayesian Equilibrium. Fixing policy assignment  $a$ , then, regardless of whether communication is private or public, there may be multiple equilibria  $(\mathbf{m}, \mathbf{y})$ . For example, the strategy profile where all players “babble” is always an equilibrium.

In distinguishing between equilibria our approach is normative. We seek to define the optimal assignment of decision-making authority given the endogenous communication structure within the majority in parliament. In doing so we rank the welfare of different assignments and the associated communication structures that emerge and assume that politicians are always able to coordinate on the equilibria  $(\mathbf{m}, \mathbf{y})$  that maximize equilibrium welfare. Our notion of welfare is ex-ante Utilitarian. Hence equilibrium welfare solves

$$\mathcal{W}(\mathbf{m}, \mathbf{y}) = - \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} E[(\hat{y}_k - \theta_k - b_i)^2].$$

However, for some of our results, we can invoke the weaker principle of Pareto optimality. Whilst our model focuses on communication of information, and so captures the advisory role of a Parliament, our normative framework captures the essence of parliamentary consent to the allocation of decision-making authority and thus the formation of the executive. In the following section we explore the forces that affect the optimal assignment of authority.

## 5. TWO FORCES BEHIND AUTHORITY ASSIGNMENT: MODERATION AND INFORMATION

We begin the analysis with a fundamental result which holds irrespective of whether information is policy specific or about a common state, and of whether information is transmitted publicly or privately to decisionmakers. We show that the optimal assignment of executive authority involves trading off: (i) the ideological moderation of those who exercise authority, and (ii) their ability to elicit information from other party politicians.

In order to formalize this insight, we first say that a politician  $j$ 's *moderation* is  $|b_j - \sum_{i \in \mathcal{I}} b_i / I|$ , the distance between  $b_j$  and the average ideology  $\sum_{i \in \mathcal{I}} b_i / I$ . We note that politicians' moderation does not depend on the assignment  $a$ , nor on the equilibrium  $(\mathbf{m}, \mathbf{y})$ . Second, we let  $d_{j,k}(\mathbf{m})$  denote politician  $j$ 's *information* on the state  $\theta_k$  given the equilibrium  $(\mathbf{m}, \mathbf{y})$ . Specifically,  $d_{j,k}(\mathbf{m})$  consists in the number of signals on  $\theta_k$  held by  $j$ , including her own, at the moment she makes her choice.

In the model specification with policy specific knowledge, each politician  $j$  may hold at most one signal on each  $\theta_k$ , either because  $s_k$  is her own signal ( $j = k$ ), or because  $s_k$  was communicated by  $k$  to  $j$  given the equilibrium communication structure  $\mathbf{c}(\mathbf{m})$ . In a specification with common value information, instead, each politician's information coincides with the number of politicians communicating truthfully with her, plus her own signal.

Armed with these definitions, given an assignment  $a$  and an equilibrium  $(\mathbf{m}, \mathbf{y})$ , we prove in the Appendix that the equilibrium ex-ante welfare  $\mathcal{W}(\mathbf{m}, \mathbf{y})$  can be rewritten as:

$$(2) \quad \mathcal{W}(\mathbf{m}, \mathbf{y}) = - \underbrace{\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} (b_i - b_{a(k)})^2}_{\text{aggregate ideological loss}} - \underbrace{\sum_{k \in \mathcal{K}} \frac{I}{6[d_{a(k),k}(\mathbf{m}) + 2]}}_{\text{aggregate residual variance}}.$$

Expression 2 decomposes the welfare function into two elements: aggregate ideological loss and the aggregate residual variance of the politicians' decisions.<sup>6</sup> Thus, in determining which assignment  $a$  maximizes welfare, we take into account each politicians' moderation and her information: assigning any task  $k$  to moderate politicians reduces the ideological loss  $\sum_{i \in \mathcal{I}} (b_{a(k)} - b_i)^2 / I$ , as their bias  $b_{a(k)}$  is closer to the average bias  $\sum_{i=1}^I b_i / I$ ; but at the same time, choosing an assignment  $a$  where the decision makers are well informed in the welfare-maximizing equilibria  $(\mathbf{m}, \mathbf{y})$  reduces the aggregate residual variance  $\sum_{k \in \mathcal{K}} [6(d_{a(k),k}(\mathbf{m}) + 2)]^{-1}$ .

<sup>6</sup>Note that, statistically, the residual variance may be interpreted as the inverse of the precision of the politicians' decisions.

We have proved the following result.

**Proposition 1.** *The optimal assignment of decision-making authority  $a$  is determined by the politicians' moderation, and by the information that they hold in equilibrium.*

The result in proposition 1 will prove central in what is to follow: we consider the case of private conversations of common state information and determine the optimal size, composition, and balance of the decision-making authority.

## 6. PRIVATE CONVERSATIONS IN COMMON STATE MODEL

We begin our study of the optimal assignment of decision making in an environment where underlying fundamentals are common to all policies so that politicians' information is relevant to all decisions. Initially we explore the situation where politicians communicate only in private with decision-makers. Since such audiences are private—no forum exists for executive members to formally exchange information— for now, we explicitly rule out cabinet governance. Other forms of government— ranging from full centralization to full decentralization, and including a ministry of decision-making politicians, responsible for different ranges of policy—are all possible.

We first describe the equilibrium communication structure given any policy assignment  $a$ . The characterization extends Corollary 1 of [Galeotti, Ghiglini, and Squintani \(2009\)](#) to the case of arbitrary policy assignments. We denote by  $d_j(\mathbf{m})$  the number of informative signals held by politician  $j$  in equilibrium. For future reference, for any assignment  $a$ , we write  $d_j^*(a)$  as the information  $d_j(\mathbf{m})$  associated with any welfare-maximizing equilibrium  $(\mathbf{m}, \mathbf{y})$ . When the state  $\theta$  is common across policies, and the communication is private, we prove in the Appendix that the profile  $\mathbf{m}$  is an equilibrium if and only if, whenever  $i$  is truthful to  $j$ ,

$$(3) \quad |b_i - b_j| \leq \frac{1}{2[d_j(\mathbf{m}) + 2]}.$$

An important consequence of equilibrium condition 3 is that truthful communication from politician  $i$  to minister  $j$ , is independent of the specific policy decisions assigned to  $j$  and of the possibility of communicating with any other politician  $j'$ . Furthermore, truthful communication from politician

$i$  to minister  $j$  becomes less likely with an increase in the difference between their ideological positions.<sup>7</sup>

The equilibrium characterization of communication between politicians and ministers subsumed by expression 3 implies a striking result for our study of information aggregation and assignment of authority in single-party governments.

**Proposition 2.** *Suppose that  $\theta$  is common across policies, and that communication is private. For generic ideologies  $\mathbf{b}$ , any Pareto optimal assignment involves decision-making authority being centralized to a single leader  $j$ : that is  $a(k) = j$  for all  $k$ .*

The finding that all decisions should be assigned to a single leader and, hence, executive authority should be fully centralized, follows from two different facts. First, truthful communication from politician  $i$  to minister  $j$  in equilibrium is independent of the specific policy decisions assigned to  $j$ , (or to any other politician  $j'$ ). Second, the stipulation that every politicians' information is relevant for all policies implies that politicians and policies are “interchangeable.” As a consequence of these two facts, whoever is the optimal politician to make one policy decision will also be the optimal politician to make all of them. Remarkably, this result holds with our utilitarian welfare criterion and under the weak welfare concept of Pareto optimality. In sum, with the restriction to private conversation between a politician and a minister, the optimal size of the executive is one: if politicians can coordinate on the optimal equilibrium, then leadership by a dominant Prime Minister emerges.

Having established the optimal size of the executive, we analyze its composition. An important element is a politician's relative ideological standing in the party. The policy bias of an active politician will affect the policy that is implemented directly. Moreover, her ideological position is a determinant of the amount of information she obtains before choosing her policy. Moving further we can micro-found the equilibrium information  $d_j^*(a)$ . In particular, and in thinking of  $j$ 's

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<sup>7</sup>A perhaps more surprising effect is that the possibility for  $i$  to communicate truthfully with  $j$  decreases with the information held by  $j$  in equilibrium. To see why communication from  $i$  to  $j$  is less likely to be truthful when  $j$  is well informed in equilibrium, suppose that  $b_i > b_j$ , so that  $i$ 's ideology is to the right of  $j$ 's bliss point. Suppose  $j$  is well informed and that politician  $i$  deviates from the truthful communication strategy –she reports  $\hat{m}_{ij} = 1$  when  $s_i = 0$ –then she will induce a small shift of  $j$ 's action to the right. Such a small shift in  $j$ 's action is always beneficial in expectation to  $i$ , as it brings  $j$ 's action closer to  $i$ 's (expected) bliss point. Hence, politician  $i$  will not be able to truthfully communicate the signal  $s_i = 0$ . By contrast, when  $j$  has a small number of players communicating with her, then  $i$ 's report  $\hat{m}_{ij} = 1$  moves  $j$ 's action to the right significantly, possibly beyond  $i$ 's bliss point. In this case, biasing rightwards  $j$ 's action may result in a loss for politician  $i$  and so she would prefer to report truthfully- that is, she will not deviate from the truthful communication strategy.

information as a consequence of her ideological position relative to that of the other politicians in her party, we define  $n_j$  as the ideological “neighbourhood” of  $j$ : the number of politicians whose ideology is within distance  $\bar{b}$  of her own. We calculate this directly as

$$n_j(\bar{b}) = \#\{i : |b_i - b_j| \leq \bar{b}\}$$

Using this definition, combined with welfare expression 2 and equilibrium condition 3, allows us to calculate  $d_j^*(a)$ .

**Lemma 1.** *Suppose that the state  $\theta$  is common across policies, and that communication is private. For any assignment  $a$ , and any active player  $j \in a(K)$ , the information  $d_j^*(a)$  solves the equation*

$$(4) \quad n_j \left( \frac{1}{2(d_j^*(a) + 2)} \right) = d_j^*(a)$$

We use this result to determine the distinguishing characteristics of the executive leader. The significance of the result in lemma 1 lies in the fact that, given any bias level  $\bar{b}$ , the magnitude of the ideological neighborhood  $n_j$  can be taken as an expression of how large is the set of politicians ideologically close to  $j$ . Ideologically close politicians translate into informants of  $j$ , in equilibrium, according to the expression in equation 4. Thus, politicians who have more ideologically like-minded allies in the party are better informed in equilibrium. We bring together these thoughts in the following corollary to proposition 2 and lemma 1:

**Corollary 1.** *Suppose that the state  $\theta$  is common across policies, that communication is private, and that ideologies  $\mathbf{b}$  are generic. Any optimal assignment centralizes executive authority to a single leader  $j$ . Optimal leadership requires ideological moderation: leader  $j$ 's policy should reflect the diversity of views in the party. Optimal leadership also requires knowledge of policy: leader  $j$ 's information depends on the number of close-minded allies she has, as defined by the function  $n_j$ .*

The identification of these two forces leading to optimal leader selection is, to our knowledge, completely novel both in the political science literature on leadership and executive politics, and in the game-theoretic literature on information transmission. Our analysis relates the twin elements that determine optimal leadership selection— the requirement for policy moderation, on the one

hand, with desire for informed policy on the other—to the communication structure that emerges in the equilibrium of our model.

## 7. CABINET MEETINGS IN COMMON STATE MODEL

Thus far we have considered communication via private meetings. We now study optimal assignment of decision making authority when information may be aggregated in public meetings. We allow for the existence of a cabinet that provides a forum where information between the set of active politicians is exchanged. This change to the communication environment affects the strategic calculus of information transmission: it is possible that politician  $i$  would not wish to communicate with minister  $j$  on a policy if that information is shared with minister  $j'$ ; conversely, politician  $i$  might share information with  $j$  *because* minister  $j'$  also has access to that information.

The next result characterizes the party's communication structure under any policy assignment  $a$ . The result extends Theorem 1 of [Galeotti, Ghiglini, and Squintani \(2009\)](#) to the case of arbitrary policy assignments.

**Lemma 2.** *Suppose that the state  $\theta$  is common across policies  $k$ , and that communication is public. The strategy profile  $\mathbf{m}$  is an equilibrium if and only if, whenever  $i$  is truthful,*

$$(5) \quad \left| b_i - \sum_{j \neq i} b_j \gamma_j(\mathbf{m}) \right| \leq \sum_{j \neq i} \frac{\gamma_j(\mathbf{m})}{2[d_j(\mathbf{m}) + 2]},$$

where for every  $j \neq i$ ,

$$\gamma_j(\mathbf{m}) \equiv \frac{a_j/[d_j(\mathbf{m}) + 2]}{\sum_{j' \neq i} a_{j'}/[d_{j'}(\mathbf{m}) + 2]}.$$

When communication is public, the set of active politicians is equivalent to the Cabinet. Intuitively, each politician  $i$ 's willingness to communicate with a member of the Cabinet depends on a weighted average of their ideologies. The specific weights are inversely related to the equilibrium information of each politician. Analyzing them reveals that, in contrast to the earlier case, truthful communication from politician  $i$  to minister  $j$  in equilibrium depends upon the policy assignment. Thus the characterization of the communication structure given by [Lemma 2](#) implies that our earlier result in [proposition 2](#)—namely that private conversation leads to fully centralized authority—can be reverted once we allow for public meetings. Then formal power-sharing agreements in a

cabinet may be optimal. We illustrate this possibility with a simple example with 4 politicians and a generic set of biases.

**Example 1.**  $\mathcal{I} = 1, 2, 3, 4$ , with  $k = 4$ . Biases are  $b_1 = -\beta$ ,  $b_2 = \varepsilon$ ,  $b_3 = \beta$ , and  $b_4 = 2\beta$ , where  $\varepsilon$  is a positive quantity smaller than  $\beta$ .<sup>8</sup> We compare four assignments, full decentralization, leadership by politician 2 (the most moderate politician), and two forms of power sharing agreements between politicians 2 and 3: in the symmetric power-sharing agreement, politicians 2 and 3 make two decisions each; in the asymmetric power-sharing agreement, politician 2 makes 3 choices, and 3 makes one choice.

The analysis requires calculating the welfare maximizing equilibria for each one of the four assignments, and then comparing welfare across assignments. Its details are relegated to the Appendix. Here, we note that taking the limit for vanishing  $\varepsilon > 0$  the following observations obtain. First, for  $\beta < 1/24$ , all players are fully informed under any of the four considered assignments; at the same time, for  $\beta > 1/12$ , there is no truthful communication regardless of the assignment; in both cases the optimal assignment entails selecting the most moderate politician 2 as the unique leader. Second, for  $\beta \in (1/24, 1/21)$ , politician 1 and 4 are willing to communicate truthfully if they are under any power sharing agreement, but politician 4 is not willing to share information if politician 2 is the single leader. Third, for  $\beta \in (1/21, 1/18)$ , players 1 and 4 are both willing to talk publicly only when the symmetric power sharing agreement is in place. Finally, for  $\beta \in (1/24, 1/18)$ , there is no advantage from assigning any choice to player 3 instead of player 2. Our result is summarized as follows.

**Result 1.** *Suppose that  $b_1 = -\beta$ ,  $b_2 = \varepsilon$ ,  $b_3 = \beta$ , and  $b_4 = 2\beta$ , and compare leadership by 2, full decentralization, and power sharing agreements between 2 and 3, under public communication of information with common state. As  $\varepsilon$  goes to zero the following holds: For  $\beta < 1/24$  or  $\beta > 1/18$ , it is optimal to select 2 as the leader; For  $\beta \in (1/24, 1/21)$ , the optimal assignment is the asymmetric power sharing agreement of 2 and 3; For  $\beta \in (1/21, 1/18)$ , the optimal assignment is the asymmetric power sharing agreement where 2 makes 3 choices, and 3 makes one choice.*

The fact that full authority centralization is always optimal when conversations are private though not necessarily when there are public meetings, together with the observation that private and

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<sup>8</sup>When  $\varepsilon = 0$  there is a multiplicity of optimal allocations, which is not generic.

public communication equilibria coincide when all authority is granted to a single leader, provides a striking result: cabinet government Pareto dominates ministerial government. This result is one of the main findings of this paper. We stress that it holds independently of whether private conversations can be used to buttress cabinet deliberations. The above argument is, evidently, conclusive when private conversations are ruled out. To assess the opposite case, note that private conversation may always involve babbling in equilibrium. Then, because we always select the Pareto optimal equilibrium of any communication game, it immediately follows that the argument developed above holds also when cabinet discussion may be supplemented with a private exchange of views between policy-makers. We state our finding formally:

**Proposition 3.** *Suppose that the state  $\theta$  is common across policies  $k$ . For generic ideologies  $\mathbf{b}$ , the optimal assignment of decision-making authority when communication is public Pareto dominates any assignments with private conversation. Cabinet government Pareto dominates ministerial government, regardless of whether private conversations can be used to supplement cabinet meetings or not.*

Proposition 3 bears important consequences for optimal executive structure. Recall the two features that describe cabinet governance: under individual ministerial responsibility decisions are taken by individual ministers; under collective responsibility the policies implemented by a minister are government policy. A requirement for collective ministerial responsibility is that information relevant to the decision is shared by Cabinet. Our result shows that if the Parliamentary majority can assign authority optimally, and politicians coordinate on the most efficient equilibria, then imposing a cabinet structure to the executive— a public meeting at a designated time and place where ministers provide the information relevant to their decisions— induces a welfare improvement over other forms of executive governance. In particular, Cabinet government Pareto dominates what we term ministerial government: a system of government where individual ministers implement policy but are not bound by collective responsibility to share policy relevant information.

## 8. OPTIMAL CABINET DESIGN

Proposition 3 establishes that cabinet government Pareto dominates ministerial government, but does not provide specific insights to the properties of the optimal assignment of authority within

a cabinet. We address this issue in three ways: we first characterize the optimal assignment of authority in a small legislature composed of three politicians; then we provide general results for large legislatures; before finally we conclude the section by presenting simulations for the intermediate case of  $I = 7$  politicians.

**8.1. Optimal Assignment in a 3 Member Parliament.** We begin with the complete analysis for  $I = 3$  politicians, for which we identify a rich characterization of the optimal assignments. Most importantly, we show that, for some ideology distributions, full decentralization is optimal in equilibrium. By contrast, later on, we will surprisingly show that this assignment is never be optimal with policy specific information.

**Example 2.** Suppose  $I = \{1, 2, 3\}$ , and let  $\Delta_1 = b_2 - b_1$  and  $\Delta_2 = b_3 - b_2$ . The optimal assignment of authorities for different values of  $\Delta_1$  and  $\Delta_2$  is illustrated in Figure 1, which we now explain. Without loss of generality, we assume in the discussion that  $\Delta_1 \leq \Delta_2$ .

Despite the fact that the set of possible assignments is large, conceptually simple, but tedious calculations, establish that only one of the following three assignments is optimal  $\{(030), (111), (120)\}$ . For example, regardless of  $\Delta_1$  and  $\Delta_2$ , the assignment (030) always dominates the assignment (300). Indeed, the incentive of politician 1 to share her information under assignment (030) is the same as the incentive of politician 2 under assignment (300). However, when the assignment is (030) politician 3 will have a higher incentive to talk publicly than when the assignment is (300).

Under assignment (030), there is an equilibrium where politician 1 and politician 3 truthfully communicate if and only if  $\Delta_2 \leq 1/10$ . Clearly, when this is the case assignment (030) is the optimal one. When both  $\Delta_2 > 1/10$  and  $\Delta_1 > 1/10$ , no communication is possible under any assignment, and, of course, the optimal assignment is (030). So, we hereafter consider the case in which  $\Delta_2 > 1/10$  and  $\Delta_1 < 1/10$ . In this case, under assignment (030) in the optimal equilibrium of the communication game only politician 1 shares his information. Simple algebra shows that, under assignment (120), politician 2 and politician 3 are willing to communicate truthfully as long as

$$(6) \quad \Delta_2 \leq \frac{33}{280} - \frac{2}{7}\Delta_1.$$

So, when  $(\Delta_1, \Delta_2)$  satisfies condition 6 and  $\Delta_2 \geq 1/10$ , assignment (120) yields a more informed choice, on average, than assignment (030). Plain calculations reveal that this results in a higher aggregate welfare than under the assignment in which all decisions are made by the most moderate politician.

Turning to assignment (111), our calculations establish that this assignment, when supporting an equilibrium in which politicians 1 and 2 communicate truthfully, yields a higher welfare than assignment (030) when  $\Delta_2 \geq 1/10$  and

$$(7) \quad \Delta_2 \leq \frac{\sqrt{10}\sqrt{120\Delta_1^2 + 1}}{20} - 2\Delta_1.$$

Simple calculations show that in region B, delimited by condition 7 and by the complement of condition 6, indeed, assignment (111) supports an equilibrium in which politicians 1 and 2 share their information. Finally, note that when  $(\Delta_1, \Delta_2)$  satisfies both condition 7 and condition 6, the assignment (120) dominates assignment (111) because, although both aggregate the same amount of information, the former produces a lower aggregate ideological loss.

These observations explain Figure 1. In region A, delimited by condition 6 and by  $\Delta_2 > 1/10$ , the optimal assignment is (120). In region B, instead, the optimal assignment is (111) and politician 1 and politician 2 share their information truthfully. In all other cases, the optimal assignment is (030). We conclude the example by summarizing the discussion as follows:

**Result 2.** *Suppose that there are  $I = 3$  politicians, and let  $\Delta_1 = b_2 - b_1$  and  $\Delta_2 = b_3 - b_2$ . The optimal authority assignment is as illustrated in Figure 1. In region A, politicians 1 and 2 share authority: the most moderate politicians makes 2 decisions, and 1 makes one decision. In region B, full decentralization is optimal. For all remaining values of  $\Delta_1$  and  $\Delta_2$ , all authority is centralized to the median politician.*

**8.2. Optimal Assignment in a Large Parliament.** The characterization for the three player case shows that optimal executives in small legislatures may yield a fairly rich characterization. By contrast, as we now show, in the limit— as the number of politicians becomes large— all decision making authority should be concentrated to politicians who are ideologically close to the most moderate one.

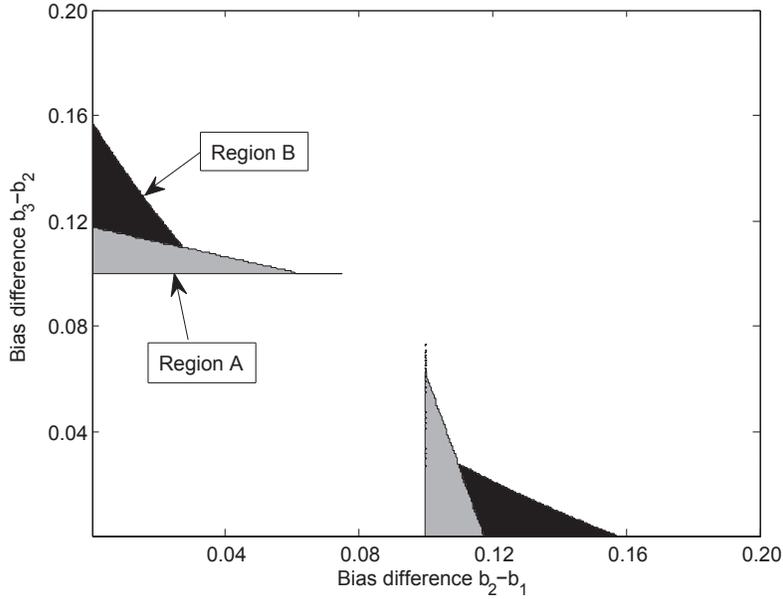


FIGURE 1. Optimal Executive Structure with 3 members.

**Proposition 4.** *Suppose that biases  $b_j$ ,  $j = 1, \dots, I$  are i.i.d. and drawn from a distribution with connected support. For every small  $\delta > 0$ , there exists a possibly large  $I_\delta > 0$  so that for all  $I > I_\delta$ , the fraction of decisions in the optimal assignment that are not concentrated to politicians with biases  $b$  such that  $|b - m_I| < \delta$  is smaller than  $\delta$ , where  $m_I = \arg \min_{i=1 \dots I} \left| b_i - \frac{1}{I} \sum_{j=1}^I b_j \right|$ .*

The proof of Proposition 4 consists of two parts. First, we show that when all the decisions are allocated to a single politician  $i$ , then as the legislature becomes large politician  $i$  becomes fully informed. Second, we compare the case in which all decisions are allocated to the most moderate politician with the case in which some of these decisions are allocated to a politician with a less moderate ideology. We show that as the legislature becomes large the aggregate residual variance obtained in each of the two assignments vanishes, whereas the difference between the aggregate ideological loss of the assignment in which decision making is shared and the centralized assignment is bounded from below.

**8.3. Cabinet Simulations.** We have so far seen that optimal executives in small legislatures can yield a rich characterization, whereas large legislature are characterized by high concentration of authority to politicians that are ideologically close to the most moderate one. To conclude our exploration of optimal decision-making authority assignments in cabinet governments, we run simulations for a 7 member parliament in which players' biases are independent and identically distributed according to a skew normal distribution, a distribution chosen for tractability. Skew normal distributions depend on three parameters which are related with the three usual moments; mean  $\mu$ , variance  $\sigma^2$  and skewness  $\gamma$ , where  $\gamma$  controls the asymmetry of the sampled distributions

TABLE 1. The Average Number of Decisions made by the Executive Leader

|                     | $\gamma = 0$ | $\gamma = 1/4$ | $\gamma = 1/2$ | $\gamma = 3/4$ | $\gamma = 1$ |
|---------------------|--------------|----------------|----------------|----------------|--------------|
| $\sigma^2 = 10$     | 7.00         | 7.00           | 7.00           | 6.99           | 7.00         |
| $\sigma^2 = 1$      | 6.91         | 6.93           | 6.89           | 6.91           | 6.88         |
| $\sigma^2 = 0.1$    | 6.17         | 6.21           | 6.18           | 6.16           | 6.21         |
| $\sigma^2 = 0.01$   | 5.58         | 5.51           | 5.53           | 5.68           | 5.67         |
| $\sigma^2 = 0.001$  | 6.35         | 6.35           | 6.37           | 6.49           | 6.35         |
| $\sigma^2 = 0.0001$ | 7.00         | 7.00           | 7.00           | 7.00           | 7.00         |

TABLE 2. Frequency with which the Executive Leader makes all Decisions

|                     | $\gamma = 0$ | $\gamma = 1/4$ | $\gamma = 1/2$ | $\gamma = 3/4$ | $\gamma = 1$ |
|---------------------|--------------|----------------|----------------|----------------|--------------|
| $\sigma^2 = 10$     | 1.00         | 1.00           | 1.00           | 0.99           | 1.00         |
| $\sigma^2 = 1$      | 0.95         | 0.96           | 0.93           | 0.95           | 0.92         |
| $\sigma^2 = 0.1$    | 0.57         | 0.60           | 0.58           | 0.61           | 0.62         |
| $\sigma^2 = 0.01$   | 0.40         | 0.36           | 0.36           | 0.41           | 0.45         |
| $\sigma^2 = 0.001$  | 0.70         | 0.70           | 0.74           | 0.78           | 0.69         |
| $\sigma^2 = 0.0001$ | 1.00         | 1.00           | 1.00           | 1.00           | 1.00         |

of ideology draws and  $\sigma$  determines the concentration of such sampled distributions draws. The normal distribution is obtained as a special case when  $\gamma = 0$ , whereas the most extreme skewness is for  $\gamma = 1$ . Because only difference in ideologies matter for our characterization, we can normalize  $\mu$  to zero, without loss of generality.

We calculate two statistics that capture the degree of centralization of authority: (i) the average number of decisions allocated to the executive leader – the individual who makes the most decisions; and (ii) the frequency of draws for which a single leader makes all decisions in a cabinet environment. The results shown in table 1 and table 2 confirm a general tendency towards centralized authority, which have been described in large legislatures by Proposition 4 and in small legislatures in example 1. In fact, the average number of decisions made by the leader ranges from 79% to 100%. Interestingly, the fraction of decisions assigned to the leader is U-shaped in the variance of the distribution, and this holds independently of the asymmetry of the distribution, or skewness. This result is consistent with the characterization of optimal assignment in the three-player case. There, for a fixed  $\Delta_1$ , an increase in  $\Delta_2$  (which represents an increase in dispersion) changes the optimal assignment from full centralization to share of authority before eventually reverting the

optimal assignment back to full centralization. Finally, allocating all actions to a single leader is often suboptimal: the frequency with which a single leader is chosen to implement *all* policy decisions may be below 50%. An implication is that in most cases centralization of authority in a multi-member cabinet is Pareto superior to other executive forms.

## 9. POLICY SPECIFIC INFORMATION

This section studies optimal assignment of decision making when each politician's information is policy specific. We assume that for each policy  $k$  there is only one expert politician  $i(k)$  who receives a signal about  $\theta_k$ . We begin by characterizing the equilibrium communication in the Parliament with dispersed expertise.

**Lemma 3.** *Suppose that information is policy specific. Under both private and public communication, the profile  $(\mathbf{m}, \mathbf{y})$  is an equilibrium if and only if, whenever politician  $i(k)$  is truthful to  $a(k) \neq i(k)$ ,*

$$|b_{i(k)} - b_{a(k)}| \leq 1/6.$$

Since each politician has only one signal and that signal is informative of only one policy decision, the amount of information held by politician  $a(k) \neq i(k)$  depends only on whether  $i(k)$  is truthful or not. Hence, whether  $i(k)$  is truthful (or not) does not depend on the communication strategy of any other politician. Further, because each politician is informed on one policy only, and this policy may be assigned to a single policy maker, private and public communication trivially coincide.

This characterization of information transmission bears a number of implications. The possibility that a politician  $i(k)$  truthfully communicates her signal to the minister  $a(k)$  to whom decision  $k$  is assigned is independent of any other assignment. Hence, for all choices  $k$ , the optimal assignment  $a(k)$  can be selected independently of other assignments. The optimal assignment is to allocate decision  $k$  to the politician  $j$  who maximizes:

$$-\sum_{i=1}^I \frac{(b_j - b_i)^2}{I} - \frac{1}{6(d_{j,k}(\mathbf{m}) + 2)},$$

where  $d_{j,k}(\mathbf{m}) = 1$  if  $|b_{i(k)} - b_j| \leq 1/6$  and  $d_{j,k}(\mathbf{m}) = 0$ , otherwise.

Simplifying the above expression, and using Lemma 3, we see that the optimal selection of  $a(k)$  takes a simple form when information is policy specific: policy decision  $k$  should be assigned to

either the most moderate politician  $m^* = \arg \min_m \left| b_m - \sum_{i=1}^I b_i / I \right|$ , or to the most moderate politician  $m(k)$  informed of  $k$ , i.e., to  $m(k) = \arg \min_{m: |b_m - b_{i(k)}| \leq 1/6} \left| b_m - \sum_{i=1}^I b_i / I \right|$ , depending on whether

$$(8) \quad \sum_{i=1}^I \frac{(b_i - b_{m(k)})^2}{I} - \sum_{i=1}^I \frac{(b_i - b_{m^*})^2}{I} < (>) \frac{1}{36}.$$

Because for any  $j$ , the quantity  $\sum_{i=1}^I (b_i - b_j)^2 / I$  is the average ideological loss, whereas the information gain is  $1/36$ , we may summarize our analysis as follows.

**Proposition 5.** *When information is policy specific, each decision  $k$  is optimally assigned to either the most moderate informed politician  $m(k)$  or to the most moderate one  $m^*$ , depending on whether the difference in average ideological loss is smaller or greater than the informational gain.*

Armed with the above characterization, we are now ready to deliver the most important result in this section. Whilst policy specific information might lead one to believe that a fully decentralized cabinet may be optimal, we now show that this is never the case. This result is even more surprising because in the common state case, in fact, full decentralization may be optimal in some parameter range, as we have proved in the analysis for  $I = 3$  politicians (Example 2).

**Proposition 6.** *Despite policy specific information, full decentralization is never optimal for generic ideologies  $\mathbf{b}$ .*

A full proof of this proposition is provided in the appendix, here we convey the main mathematical intuition behind the result. Note that when there are politicians  $i, j$  with  $|b_i - b_j| < 1/6$ , then  $i$  truthfully communicates to  $j$  and vice-versa. For generic ideologies  $\mathbf{b}$ , either  $i$  or  $j$  is closest to the average ideology  $\sum_{i=1}^I b_i / I$ , and hence either  $i$  improves welfare by taking  $j$ 's decision, or vice-versa. So, consider that  $b_i - b_{i-1} \geq 1/6$ , for all  $i$ , and no communication takes place. Because of the concavity of the payoff function, spreading biases makes extremism less favorable to the pool of politicians. Hence, we take  $b_i - b_{i-1} = 1/6$  for all  $i$ , and show that assigning choice 1 to politician 1 yields lower welfare than assigning it to a moderate politician.

In the remainder of this section we investigate in some detail the features of optimal assignment of decisions in legislatures where information is policy specific.

**9.1. Optimal Assignment in a Large Parliament.** We first consider the case of large legislatures and ask how the degree of centralisation in the assignment of policies depends on the underlying distribution of biases. Suppose biases  $\{b_1, \dots, b_I\}$  are i.i.d. and drawn from a single-peaked distribution  $f$  with mean  $\mu$ . As the size of the legislatures grows large the bias  $b_{m^*}$  of the most moderate politician  $m^*$  tends to  $\mu$ . In this case, Proposition 5 takes the following simple form: In the limit, as  $I$  goes to infinity, each decision  $k$  is optimally assigned to politician  $a(k)$  according to the following rule. For all  $k$  such that  $b_{i(k)} < \mu - 1/3$  or  $\mu - 1/6 < b_{i(k)} < \mu + 1/6$  or  $b_{i(k)} > \mu + 1/3$ , it is the case that  $a(k) = m^*$ . For all  $k$  such that  $\mu - 1/3 < b_{i(k)} < \mu - 1/6$ ,  $a(k) = i(k) + 1/6$ , and finally, for all  $\mu + 1/6 < b_{i(k)} < \mu + 1/3$ ,  $a(k) = i(k) - 1/6$ . Hence, as the legislature grows large the expected fraction of policies allocated to the most moderate politician  $m^*$  is

$$\Pr(a(k) = m^*) = \int_{-\infty}^{\mu-1/3} f(k) dk + \int_{\mu-1/6}^{\mu+1/6} f(k) dk + \int_{\mu+1/3}^{+\infty} f(k) dk.$$

Despite the existence of politicians with policy specific expertise, all policies should be assigned to the most moderate politician unless the politician who is an expert on the specific policy has an “intermediate” bias: her views are neither too close to the most moderate politician, nor too extreme relative to the average view held within the parliamentary majority. Thus the fraction of policies allocated to the most moderate politician will depend on the fraction of intermediate politicians in the governing majority, which, in turn, depends on the dispersion of ideologies.

The next proposition shows that the fraction of policies allocated to the most moderate politician is not monotonic in the dispersion of policy expertise. Intuitively, it converges to one when the ideology distribution becomes sufficiently concentrated; under mild regularity conditions, it converges to one also when the ideology distribution becomes very dispersed.

**Proposition 7.** *Assume biases  $\{b_1, \dots, b_I\}$  are i.i.d. and drawn from a single peaked distribution  $f$  with finite variance  $\sigma^2$ , and let  $I$  grow large. As  $\sigma^2$  goes to zero, or as  $\sigma^2$  goes to infinity, as long as in this case the distribution’s peak converges to zero, the fraction of decisions allocated to the most moderate agent converges to one. Hence, the fraction of policies allocated to the most moderate politician is non-monotonic in the ideology dispersion.*

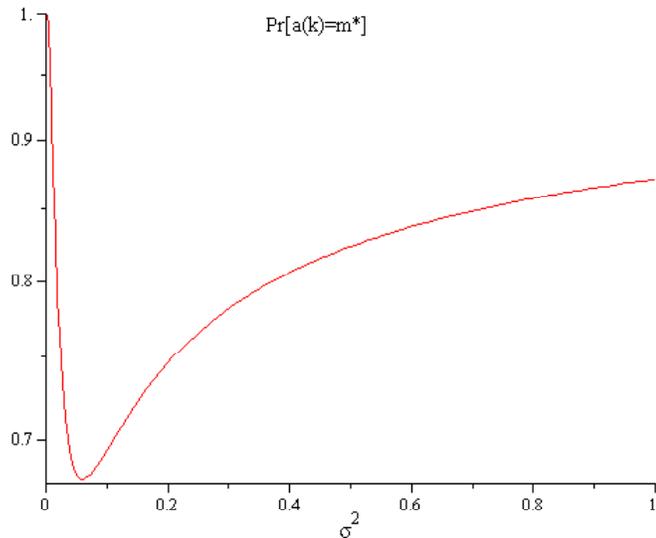


FIGURE 2. Expected fraction of policies allocated to  $m^*$  for large legislatures.

The proof of proposition 7 is based on simple intuitions. Recall that the only decisions that should not be given to the most moderate politician are the ones where “experts” have ‘intermediate’ ideology: they are neither too moderate, nor too extreme. When the variance of the ideology distribution converges to zero, the fraction of politicians with intermediate ideology becomes negligible. When the variance of the ideology distribution becomes infinite, as long as the distribution’s peak converges to zero, all the probability mass of the ideology distribution is pushed to the tails, and thus the fraction of politicians with intermediate ideology also becomes negligible.

We show in the Appendix that the skew normal distribution (which we use for simulations) satisfies these regularity conditions. Furthermore, numerical analysis shows that the fraction of decisions assigned to the most moderate politician is U-shaped in the ideology distribution dispersion. Figure 2 illustrates how this quantity changes with an increase in the variance, keeping constant the mean, in the case of the normal distribution. A similar picture is obtained for any level of skewness of the skew normal distribution.

**9.2. Cabinet Simulations.** Moving beyond the results for large legislatures, we now discuss numerical results obtained for legislatures with  $I = 7$  politicians. The simulation shown in Table 3 and 4 report the leader’s average number of assigned decisions and the frequency with which the executive leader makes decisions, when information is policy specific. The results show that the centralization of executive authority is not smaller (in fact, usually, it is larger) than in the (common-state) case where expertise is evenly distributed, for given parameter values,  $\gamma$  and  $\sigma$  of

the skew normal distribution. Thus, whilst we have uncovered a rich equilibrium behavior that allows for both single leadership and power sharing agreements in the form of a cabinet, surprisingly, we have found that the optimal decision-making authority assignment is no more decentralized than in the common-state case. Furthermore, the fraction of decisions made by the most moderate politician is non-monotonic in the dispersion of politicians' ideologies. This is consistent with the result in Proposition 7 derived for large legislatures.

We conclude by referring back to our motivating historical example. Recall that, according to Cox (1987), centralization of authority in Victorian England can be explained as due to the asymmetric distribution of expertise in the Parliament. Our information aggregation perspective reveals that the optimal assignment of decision-making authority involves centralization, sometimes to a Cabinet, other times to a unique individual minister, and that the forces that drive this process do not crucially depend on the distribution of policy expertise in the Parliament.

TABLE 3. Average Number of Decisions Assigned to the Executive Leader when Information is Policy-specific

|                     | $\gamma = 0$ | $\gamma = 1/4$ | $\gamma = 1/2$ | $\gamma = 3/4$ | $\gamma = 1$ |
|---------------------|--------------|----------------|----------------|----------------|--------------|
| $\sigma^2 = 10$     | 6.96         | 6.95           | 6.97           | 6.97           | 6.97         |
| $\sigma^2 = 1$      | 6.70         | 6.73           | 6.69           | 6.81           | 6.74         |
| $\sigma^2 = 0.1$    | 5.97         | 5.90           | 5.90           | 5.86           | 5.75         |
| $\sigma^2 = 0.01$   | 6.59         | 6.54           | 6.53           | 6.57           | 6.55         |
| $\sigma^2 = 0.001$  | 7.00         | 7.00           | 7.00           | 7.00           | 7.00         |
| $\sigma^2 = 0.0001$ | 7.00         | 7.00           | 7.00           | 7.00           | 7.00         |

TABLE 4. Frequency with which the Executive Leader makes all Decisions

|                     | $\gamma = 0$ | $\gamma = 1/4$ | $\gamma = 1/2$ | $\gamma = 3/4$ | $\gamma = 1$ |
|---------------------|--------------|----------------|----------------|----------------|--------------|
| $\sigma^2 = 10$     | 0.97         | 0.96           | 0.97           | 0.97           | 0.97         |
| $\sigma^2 = 1$      | 0.78         | 0.82           | 0.80           | 0.85           | 0.83         |
| $\sigma^2 = 0.1$    | 0.38         | 0.39           | 0.39           | 0.37           | 0.29         |
| $\sigma^2 = 0.01$   | 0.66         | 0.64           | 0.64           | 0.64           | 0.66         |
| $\sigma^2 = 0.001$  | 1.00         | 1.00           | 1.00           | 1.00           | 1.00         |
| $\sigma^2 = 0.0001$ | 1.00         | 1.00           | 1.00           | 1.00           | 1.00         |

## 10. CONCLUDING DISCUSSION

Democratic legitimacy rests upon the consent given to those who exercise decision-making authority. A key empirical regularity is the centralization of such authority; in parliamentary government, in particular, such centralization takes the striking form of a cabinet government, often dominated by a Prime Minister. We have developed a novel model of collective assignment of decision-making authority within a Parliamentary majority where politicians are privately informed about policies and are ideological differentiated. We analyzed the optimal assignment of authority when politicians strategically communicate this information. Our results provide normative justification for centralized authority whether information is specific to each policy or common to all policies under consideration. We also explored the effects of cabinet deliberations on strategic communication and showed that cabinet government Pareto dominates ministerial government. We find that the key forces that drive the selection of the (optimal) executive leader are ideological moderation and the number of close-minded allies she has. The latter effect is due to the increased ability of politicians with a high concentration of ideologically close party allies to aggregate information.

Our information analysis provides a new framework for understanding the existence of assemblies in which diverse preferences and strong factional alliances sit alongside centralized executive authority. Indeed, our paper provides an alternative normative framework for understanding historical episodes such as the establishment of an all-powerful executive that fused legislative and executive powers in Victorian England. An important, and till now, unanswered part of that historical puzzle is why the need for centralization gave rise to Cabinet government. We provide normative foundations from an information aggregation perspective for this particular executive.

Whilst our work establishes a normative benchmark for evaluating the assignment of decision-making authority in parliamentary government, in practice the assignment of decision-making rights is carried out by a Prime Minister; thus our analysis reveals what a welfare maximizing premier would do. Extensions will consider the assignment of authority as part of the Prime Minister's strategic plan, when her objectives may conflict with those of members of her party.

Here we have considered a Parliament with a given ideological profile. A natural extension is to consider how centralized authority affects the actions of party elites and voters whose actions jointly determine the ideological composition of the assembly. A further avenue of research is how

the degree of centralization of decision-making authority responds to party control over nomination of the members of Parliament.

These substantive applications can be approached within the current modeling framework. One further line of enquiry can be pursued, which would involve more extensive modifications of the model. Here we have assumed that the parliamentary majority assigns “decision-making authority” to ministers, having in mind the fusion of legislative and executive powers found in many parliamentary democracies. Our model could be modified so that the Parliamentary majority nominates agenda setters whose proposal needs then to be formally approved by the Parliament.

## 11. APPENDIX

### Equilibrium beliefs.

In our model a politicians’ equilibrium updating is based on the standard Beta-binomial model. Suppose that a politician  $i$  holds  $n$  bits of information, i.e. she holds the private signal  $s_i$  and  $n - 1$  politicians truthfully reveal their signal to her. The probability that  $l$  out of such  $n$  signals equal one, conditional on  $\theta$  is

$$f(l|\theta, n) = \frac{n!}{l!(n-l)!} \theta^l (1-\theta)^{(n-l)}.$$

Hence, politician  $i$ ’s posterior is

$$f(\theta|l, n) = \frac{(n+1)!}{l!(n-l)!} \theta^l (1-\theta)^{(n-l)},$$

the expected value is

$$E(\theta|l, n) = \frac{l+1}{n+2},$$

and the variance is

$$V(\theta|l, n) = \frac{(l+1)(n-l+1)}{(n+2)^2(n+3)}.$$

**Derivation of equilibrium welfare, expression 2.** Assume  $(\mathbf{m}, \mathbf{y})$  is an equilibrium. The ex-ante expected utility of each player  $i$  is:

$$\begin{aligned}
Eu_i(\mathbf{m}, \mathbf{y}) &= -E \left[ \sum_{k=1}^K (y_k - \theta - b_i)^2; (\mathbf{m}, \mathbf{y}) \right] \\
&= -\sum_{k=1}^K E \left[ (y^{a(k)} - \theta - b_i)^2; (\mathbf{m}, \mathbf{y}) \right] \\
&= -\sum_{k=1}^K E \left[ (b_{a(k)} + E[\theta | \Omega_{a(k)}] - \theta - b_i)^2; \mathbf{m} \right]
\end{aligned}$$

where  $\Omega_{a(k)}$  denotes the equilibrium information of player  $a(k)$ . Hence

$$\begin{aligned}
Eu_i(\mathbf{m}, \mathbf{y}) &= -\sum_{k=1}^K E \left[ (b_{a(k)} - b_i)^2 + (E[\theta | \Omega_{a(k)}] - \theta)^2 - 2(b_{a(k)} - b_i)(E[\theta | \Omega_{a(k)}] - \theta); \mathbf{m} \right] \\
&= -\sum_{k=1}^K \left[ (b_{a(k)} - b_i)^2 + E \left[ (E[\theta | \Omega_{a(k)}] - \theta)^2; \mathbf{m} \right] \right. \\
&\quad \left. - 2(b_{a(k)} - b_i)(E[E[\theta | \Omega_{a(k)}]; \mathbf{m}] - E[\theta; \mathbf{m}]) \right],
\end{aligned}$$

by the law of iterated expectations,  $E[E[\theta | \Omega_{a(k)}]; \mathbf{m}] = E[\theta; \mathbf{m}]$ , and by definition  $E \left[ (E[\theta | \Omega_{a(k)}] - \theta)^2; \mathbf{m} \right] = \sigma_k^2(\mathbf{m})$ .

Further, note that the equilibrium information  $\Omega_{a(k)}$  of player  $a(k)$  may be represented as any vector in  $\{0, 1\}^{d_{a(k)}(\mathbf{c})+1}$ . Letting  $l$  be the number of digits equal to one in any such vector, we obtain

$$\begin{aligned}
E \left[ (E[\theta | \Omega_{a(k)}] - \theta)^2; \mathbf{m} \right] &= \int_0^1 \sum_{l=0}^{d_{a(k)}(\mathbf{c})+1} (E[\theta | l, d_{a(k)}(\mathbf{c}) + 1] - \theta)^2 f(l | d_{a(k)}(\mathbf{c}) + 1, \theta) d\theta \\
&= \int_0^1 \sum_{l=0}^{d_{a(k)}(\mathbf{c})+1} (E[\theta | l, d_{a(k)}(\mathbf{c}) + 1] - \theta)^2 \frac{f(\theta | l, d_{a(k)}(\mathbf{c}) + 1)}{d_{a(k)}(\mathbf{c}) + 1 + 1} d\theta,
\end{aligned}$$

where the second equality follows from  $f(l | d_{a(k)}(\mathbf{c}) + 1, \theta) = f(\theta | l, d_{a(k)}(\mathbf{c}) + 1) / (d_{a(k)}(\mathbf{c}) + 2)$ .

Because the variance of a beta distribution of parameters  $l$  and  $d + 1$ , is

$$V(\theta | l, d + 1) = \frac{(l + 1)(d + 1 - l + 1)}{(d + 1 + 2)^2 (d + 1 + 3)},$$

we obtain:

$$\begin{aligned}
E \left[ (E [\theta | \Omega_{a(k)}] - \theta)^2; \mathbf{m} \right] &= \frac{1}{d_{a(k)}(\mathbf{c}) + 2} \left[ \sum_{l=0}^{d_{a(k)}(\mathbf{c})+1} V(\theta | l, d_{a(k)}(\mathbf{c}) + 1) \right] \\
&= \sum_{l=0}^{d_{a(k)}(\mathbf{c})+1} \frac{(l+1)(d_{a(k)}(\mathbf{c}) - l + 2)}{(d_{a(k)}(\mathbf{c}) + 2)(d_{a(k)}(\mathbf{c}) + 3)^2(d_{a(k)}(\mathbf{c}) + 4)} \\
&= \frac{1}{6(d_{a(k)}(\mathbf{c}) + 3)}.
\end{aligned}$$

■

**Proof of Proposition 1.** Fix any assignment  $a$ . Any Pareto optimal equilibrium  $(\mathbf{m}, \mathbf{y})$  maximizes the welfare

$$\mathcal{W}(\mathbf{m}, \mathbf{y}; \gamma) = - \sum_{i \in \mathcal{I}} \gamma_i \sum_{k \in \mathcal{K}} E[(y_k - \theta_k - b_i)^2 | s_i, \mathbf{m}_{\bar{N}_i, i}],$$

for some Pareto weights  $\gamma$ . Following the same steps in the derivation of expression 2 we obtain that

$$\mathcal{W}(\mathbf{m}, \mathbf{y}; \gamma) = - \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \gamma_i (b_i - b_{a(k)})^2 - \sum_{k \in \mathcal{K}} \frac{1}{6[d_{a(k), k}(\mathbf{m}) + 2]}.$$

This decomposition, together with equilibrium condition 3, imply that, as long as  $j$  is active under  $a$  the equilibrium information  $d_j^*(a)$  associated to any Pareto optimal equilibrium  $(\mathbf{m}, \mathbf{y})$  is independent of the set of policy choices  $a^{-1}(i)$  assigned to any player  $i$ , including  $j$ . Hence, choosing the Pareto-optimal assignment is equivalent to finding the index  $j$  that maximizes

$$(9) \quad - \sum_{i=1}^I \gamma_i (b_j - b_i)^2 - \frac{1}{6(d_j(\mathbf{m}) + 2)},$$

and to assigning all policy choices  $k$  to such optimal  $j$ . For generic vectors of biases  $\mathbf{b}$ , the expression (9) has a unique maximizer. ■

**Proof of Lemma 1.** Because  $n_j(\bar{b})$  is step function, increasing in  $\bar{b}$ , and  $1/[2(d+2)]$  strictly decreases in  $\bar{b}$ , whereas the identity is strictly increasing in  $d$ , there is a unique solution to equation (4). From equilibrium condition 3, we see that maximization of  $\mathcal{W}(\mathbf{m}, \mathbf{y})$  is equivalent to maximization of the equilibrium information  $d_j(\mathbf{m})$  of each active player  $j \in a(K)$ . Equilibrium condition 3 shows that the maximal information of each active player  $j \in a(K)$  can be calculated independently of the other active players' information, according to equation (4). ■

**Proof of Lemma 2 and Derivation of Expression 3.** We first prove Lemma 2 and then derive Expression 3 as a corollary. Consider any  $j \in a(\mathcal{K})$ , and suppose let  $C_j(\mathbf{c})$  be the set of players truthfully communicating with  $j$  in equilibrium, i.e. the equilibrium network neighbors of  $j$ . The equilibrium information of  $j$  is thus  $d_j = |C_j(\mathbf{c})| + 1$ , the cardinality of  $C_j(\mathbf{c})$  plus  $j$ 's signal. Consider any player  $i \in C_j(\mathbf{c})$ . Let  $s_R$  be the vector containing  $s_j$  and the (truthful) messages of all players in  $C_j(\mathbf{c})$  except  $i$ . Let also  $y_{s_R, s}^j$  be the action that  $j$  would take if he has information  $s_R$  and player  $i$  has sent signal  $s$ ; analogously,  $y_{s_R, 1-s}^j$  is the action that  $j$  would take if he has information  $s_R$  and player  $i$  has sent signal  $1 - s$ . Agent  $i$  reports truthfully signal  $s$  to a collection of agents  $J$  if and only if

$$-\sum_{j \in J} \sum_{k: a(k)=j} \int_0^1 \sum_{s_R \in \{0,1\}^{d_j}} \left[ (y_{s_R, s}^j - \theta - b_i)^2 - (y_{s_R, 1-s}^j - \theta - b_i)^2 \right] f(\theta, s_R | s) d\theta \geq 0.$$

Using the identity  $a^2 - b^2 = (a - b)(a + b)$  and simplifying, we obtain:

$$-\sum_{j \in J} \int_0^1 a_j \sum_{s_R \in \{0,1\}^{d_j}} \left[ (y_{s_R, s}^j - y_{s_R, 1-s}^j) \left( \frac{y_{s_R, s}^j + y_{s_R, 1-s}^j}{2} - (\theta + b_i) \right) \right] f(\theta, s_R | s) d\theta \geq 0.$$

Next, observing that

$$y_{s_R, s}^j = b_j + E[\theta | s_R, s],$$

we obtain

$$\begin{aligned} & -\sum_{j \in J} \int_0^1 a_j \sum_{s_R \in \{0,1\}^{d_j}} [(E[\theta + b_j | s_R, s] - E[\theta + b_j | s_R, 1 - s])] \\ & \cdot \left( \frac{E[\theta + b_j | s_R, s] + E[\theta + b_j | s_R, 1 - s]}{2} - (\theta + b_i) \right) f(\theta, s_R | s) d\theta \geq 0. \end{aligned}$$

Denote

$$\Delta(s_R, s) = E[\theta | s_R, s] - E[\theta | s_R, 1 - s].$$

Observing that:

$$f(\theta, s_R | s) = f(\theta | s_R, s) P(s_R | s),$$

and simplifying, we get:

$$-\sum_{j \in J} a_j \sum_{s_R \in \{0,1\}^{d_j}} \int_0^1 \left[ \Delta(s_R, s) \left( \frac{E[\theta|s_R, s] + E[\theta|s_R, 1-s]}{2} + b_j - b_i - \theta \right) \right] \cdot f(\theta|s_R, s) P(s_R|s) d\theta \geq 0.$$

Furthermore,

$$\int_0^1 \theta f(\theta|s_R, s) d\theta = E[\theta|s_R, s],$$

and

$$\int_0^1 P(\theta|s_R, s) E[\theta|s_R, s] d\theta = E[\theta|s_R, s],$$

because  $E[\theta|s_R, s]$  does not depend on  $\theta$ . Therefore, we obtain:

$$\begin{aligned} & -\sum_{j \in J} a_j \sum_{s_R \in \{0,1\}^{d_j}} \left[ \Delta(s_R, s) \left( \frac{E[\theta|s_R, s] + E[\theta|s_R, 1-s]}{2} + b_j - b_i - E[\theta|s_R, s] \right) \right] P(s_R|s) \\ &= -\sum_{j \in J} a_j \sum_{s_R \in \{0,1\}^{d_j}} \left[ \Delta(s_R, s) \left( -\frac{E[\theta|s_R, s] - E[\theta|s_R, 1-s]}{2} + b_j - b_i \right) \right] P(s_R|s) \geq 0. \end{aligned}$$

Now, note that:

$$\begin{aligned} \Delta(s_R, s) &= E[\theta|s_R, s] - E[\theta|s_R, 1-s] \\ &= E[\theta|l+s, d_j+1] - E[\theta|l+1-s, d_j+1] \\ &= (l+1+s)/(d_j+3) - (l+2-s)/(d_j+3) \\ &= \begin{cases} -1/(d_j+3) & \text{if } s=0 \\ 1/(d_j+3) & \text{if } s=1. \end{cases} \end{aligned}$$

where  $l$  is the number of digits equal to one in  $s_R$ . Hence, we obtain that agent  $i$  is willing to communicate to agent  $j$  the signal  $s=0$  if and only if:

$$-\sum_{j \in J} a_j \left( \frac{-1}{d_j+3} \right) \left( -\frac{-1}{2(d_j+3)} + b_j - b_i \right) \geq 0,$$

or

$$\sum_{j \in J} a_j \frac{b_j - b_i}{d_j+3} \geq -\sum_{j \in J} a_j \frac{1}{2(d_j+3)^2}$$

Note that this condition is redundant if  $\sum_{j \in J} a_j (b_j - b_i) > 0$ . On the other hand, she is willing to communicate to agent  $j$  the signal  $s = 1$  if and only if:

$$-\sum_{j \in J} a_j \left( \frac{1}{d_j + 3} \right) \left( -\frac{1}{2(d_j + 3)} + b_j - b_i \right) \geq 0,$$

or

$$\sum_{j \in J} a_j \frac{b_j - b_i}{d_j + 3} \leq \sum_{j \in J} a_j \frac{1}{2(d_j + 3)^2}.$$

Note that this condition is redundant if  $\sum_{j \in J} a_j (b_j - b_i) < 0$ . Collecting the two conditions yields:

$$(10) \quad \left| \sum_{j \in J} a_j \frac{b_j - b_i}{d_j + 3} \right| \leq \sum_{j \in J} \frac{a_j}{2(d_j + 3)^2}.$$

Rearranging condition 10 completes the proof of Lemma 2. ■

**Proof of Result 1.** Consider a cabinet of 4 politicians, with biases  $b_1 = -\beta$ ,  $b_2 = \varepsilon$ ,  $b_3 = \beta$ , and  $b_4 = 2\beta$ . We suppose that  $\varepsilon > 0$  is small, so that politician 2 is the most moderate. We compare four assignments, full decentralization, leadership by politician 2, a symmetric power-sharing agreement where politicians 2 and 3 make two decisions each, and an asymmetric power-sharing agreement where politician 2 makes 3 choices, and 3 makes one choice.

Consider leadership by politician 2, first. Using Lemma ??, we obtain that  $d_2 = 4$  if  $2\beta - \varepsilon \leq 1/12$ , i.e.,  $\beta \leq \varepsilon/2 + 1/24$ , whereas  $d_2 = 3$  if  $\beta + \varepsilon \leq 1/10$ , i.e.  $\beta \leq 1/10 - \varepsilon$ , as well as  $d_2 = 2$  if  $\beta - \varepsilon \leq 1/8$ , i.e.,  $\beta \leq 1/8 + \varepsilon$ , and  $d_2 = 1$  if  $\beta > 1/8 + \varepsilon$ .

Consider the symmetric power sharing rule. First note that, if 1 is willing to talk, then so are all other players. Hence, for  $2(\beta + \varepsilon) + 2 \cdot 2\beta \leq \frac{4}{2(3+3)}$ , i.e.,  $\beta \leq 1/18 - \varepsilon/3$ , then both  $d_2 = 4$  and  $d_3 = 4$ . Further, for  $2(2\beta - \varepsilon) + 2\beta \leq \frac{4}{2(2+3)}$ , i.e.,  $\beta \leq 1/15 + \varepsilon/3$ , then 1 does not talk, but 4 does, and so,  $d_2 = 3$  and  $d_3 = 3$ . Finally, for  $\beta - \varepsilon \leq 1/8$ , i.e.,  $\beta \leq 1/8 + \varepsilon$ , then both 2 and 3 talk to each other:  $d_2 = 2$  and  $d_3 = 2$ . Of course,  $d_2 = 1$  and  $d_3 = 1$ , if  $\beta > 1/8 + \varepsilon$ .

Hence, the symmetric power sharing rule dominates the single leader 2 on  $(\varepsilon/2 + 1/24, 1/18 - \varepsilon/3]$  in terms of information transmission. It will dominate on a subset, because of the moderation effect, but as  $\varepsilon \rightarrow 0$ , the subset converges to  $(1/24, 1/18)$ .

Consider now the asymmetric power sharing rule. In this case the condition for 1 to talk (if 4 is talking) becomes:  $\frac{3}{4}(\beta + \varepsilon) + \frac{1}{4}2\beta \leq \frac{1}{2(3+3)}$ , i.e.,  $\beta \leq 1/15 - 3\varepsilon/5$ . The condition for 4 to

talk if 1 is talking becomes,  $\frac{3}{4}(2\beta - \varepsilon) + \frac{1}{4}\beta \leq \frac{1}{2(3+3)}$ , i.e.,  $\beta \leq 1/21 + 3\varepsilon/7$ . Hence, for  $\beta \leq 1/21 + 3\varepsilon/7$ , then both  $d_2 = 4$  and  $d_3 = 4$ . Instead, the condition for 1 to talk if 4 does not talk is  $\frac{3}{4}(\beta + \varepsilon) + \frac{1}{4}2\beta \leq \frac{1}{2(2+3)}$ , i.e.,  $\beta \leq 2/25 - 3\varepsilon/5$ . And the condition for 4 to talk if 1 does not talk is  $\frac{3}{4}(2\beta - \varepsilon) + \frac{1}{4}\beta \leq \frac{1}{2(2+3)}$ , i.e.  $\beta \leq \frac{3}{7}\varepsilon + \frac{2}{35}$ . Hence, for  $\beta \leq 2/25 - 3\varepsilon/5$ , then both  $d_2 = 3$  and  $d_3 = 3$ . The condition for 2 and 3 to each other talk is  $\beta \leq 1/8 + \varepsilon$ ; in this case  $d_2 = 2$  and  $d_3 = 2$ . Again,  $d_2 = 1$  and  $d_3 = 1$ , if  $\beta > 1/8 + \varepsilon$ .

Hence, the asymmetric power sharing agreement dominates the single leader 2 informationally on  $(\varepsilon/2 + 1/24, 1/21 - 3\varepsilon/7]$ . Due to the moderation effect, it also dominates the symmetric power sharing agreement. For  $\varepsilon \rightarrow 0$ , asymmetric power sharing agreement dominates on  $(1/24, 1/21)$ .

Finally, consider full decentralization. The player who is least likely to speak publicly is 1. Given that all other players speak, he speaks if and only if  $(\beta + \varepsilon) + 2\beta + 3\beta \leq \frac{3}{2[3+3]}$  or  $\beta \leq \frac{1}{24} - \varepsilon/6$ . In this case, all players receive 3 signals,  $\frac{1}{24} = 0.041667$ . Then, if 1 does not speak, the least likely to speak is 4. This occurs if and only if  $\frac{3\beta}{3+3} + \frac{2\beta - \varepsilon + \beta}{2+3} \leq \frac{1}{2(3+3)^2} + \frac{2}{2(2+3)^2}$ , i.e. if  $\beta \leq \frac{2}{11}\varepsilon + \frac{97}{1980}$  for  $\varepsilon \rightarrow 0$ , this is close to  $\frac{97}{1980} \approx 0.04899$ . When 1 does not speak publicly, whereas 4 does, the  $d$ -distribution is: 3, 2, 2, 2; which is informationally better than the private communication to 2. But, of course, it is worse in terms of moderation... Further, decentralization is dominated by the symmetric power sharing agreements, for the range  $\beta \leq 1/18 - \varepsilon/3$ , as  $1/18 \approx 0.055556$ ; because in this range  $d_2 = 3$  and  $d_3 = 3$  for the asymmetric power sharing agreements. Then, if 1 and 4 do not speak, the least likely to speak is 3 —because 2 is more central. This occurs if and only if  $\frac{2\beta}{2+3} + \frac{\beta - \varepsilon}{1+3} + \frac{\beta}{2+3} \leq \frac{1}{2(1+3)^2} + \frac{2}{2(2+3)^2}$ , i.e.,  $\beta \leq \frac{5}{17}\varepsilon + \frac{57}{680} \approx 0.083824$ , with the distribution 2, 1, 1, 2. This is dominated by the asymmetric power sharing agreements, because for  $\beta \leq 1/10 - 3\varepsilon/5$ , i.e., essentially,  $\beta \leq 1/10$ , we have  $d_2 = 2$  and  $d_3 = 2$ . Finally, 2's condition to speak if nobody else speaks under decentralization is:  $2\beta - \varepsilon + \beta - \varepsilon + \beta + \varepsilon \leq \frac{3}{2(1+3)}$ , i.e.  $\beta \leq \frac{1}{4}\varepsilon + \frac{3}{32} \approx 0.09375$ . Because this yields the distribution 1, 0, 1, 1, we obtain that it is dominated by the asymmetric power sharing agreements. ■

**Proof of Proposition 3.** From Proposition 2, we know that all Pareto optimal assignments  $a$  under private communication of common value information entails a single leader, i.e., there is  $j$  such that  $a(k) = j$  for all  $k$ . Suppose now that communication is public, and suppose that an assignment  $a$  with a unique leader  $j$  is selected. Then, because  $\gamma_j(\mathbf{m}) = 1$  and  $\gamma_{j'}(\mathbf{m}) = 0$  for all  $j' \neq j$ , condition (5) in Lemma 2 reduces to condition (3). Hence, the set of equilibria under

private and public communication coincide under  $a$ . But because the optimal assignment under public communication  $a^*$  need not entail a single leader, the statement of the result immediately follows in the case that private conversations are ruled out under public communication. Allowing for private conversations does not change the argument, because babbling all private conversations is always possibly part of an equilibrium, and we select the optimal equilibrium in any communication game which follow the assignment and the choice of communication rule. ■

**Proof of Proposition 4** The proof of proposition 4 proceeds in two steps. The first step shows that if all decisions are allocated to a single agent, the information of this agent approaches infinity as the number of agents  $I$  goes to infinity. This is formalised in the following lemma.

**Lemma 4.** *Suppose that biases  $b_j$ ,  $j = 1, 2, \dots, I$  are i.i.d. and drawn from a distribution of connected support. If all decisions are assigned to the same politicians  $i$ , then the optimal equilibrium information  $d_i^I$  of politician  $i$  grows to infinity in probability as  $I$  becomes infinite.*

**Proof of Lemma 4.** Recall that for any  $I$ , the optimal equilibrium information  $d_i^I$  solves the condition

$$\left| \left\{ j = 1, 2, \dots, N : |b_i - b_j| \leq \frac{1}{2(d_i^I + 2)} \right\} \right| = d_i^I.$$

We now to show that, for  $d > 0$ ,

$$\lim_{I \rightarrow \infty} \Pr(d_I \leq d) = 0.$$

Note, in fact, that:

$$\begin{aligned} \Pr(d_I \leq d) &= \Pr\left(\left|\left\{j = 1, 2, \dots, N : |b_i - b_j| \leq \frac{1}{2(d+2)}\right\}\right| \leq d\right) \\ &= \Pr\left(\times_{j=1}^{I-d} \left\{b_j : |b_i - b_j| > \frac{1}{2(d+2)}\right\}\right) \\ &= \left(\Pr\left\{b_j < b_i - \frac{1}{2(d+2)}\right\} + \Pr\left\{b_j > b_i + \frac{1}{2(d+2)}\right\}\right)^{I-d}, \end{aligned}$$

and it is now immediate to see that

$$\lim_{I \rightarrow \infty} \Pr(d_I \leq d) = \lim_{I \rightarrow \infty} \left(\Pr\left\{b_j < b_i - \frac{1}{2(d+2)}\right\} + \Pr\left\{b_j > b_i + \frac{1}{2(d+2)}\right\}\right) = 0$$

This concludes the proof of Lemma 4. ■

We now turn to the second step. We compare the expected per-person per-action payoff  $W_I^{m_I}$  for assigning all decisions  $K$  to the most moderate politician  $m_I = \arg \min_i \left( b_i - \sum_{j=1}^I \frac{b_j}{I} \right)^2$ , to the payoff  $W_I^{\alpha \varepsilon}$  for assigning a fraction  $\alpha_I \geq \alpha > 0$  of the  $K$  actions, such that  $\alpha_I K$  is an integer, to a different politician  $j_I$  such that  $b_{j_I} - E[b_j] > \delta > 0$ , for all  $I$ . The remaining fraction  $1 - \alpha_I$  of actions is assigned to  $m_I$ . Hence,

$$\begin{aligned}
W_I^{m_I} - W_I^{\alpha \varepsilon} &= E \left[ \alpha_I \left( \sum_{i=1}^I \frac{(b_i - b_j)^2}{I} - \sum_{i=1}^I \frac{(b_i - b_{m_I})^2}{I} + \frac{1}{2(d_j + 2)} - \frac{1}{2(d_{m_I} + 2)} \right) \right] \\
&= \alpha_I E \left[ \sum_{i=1}^I \frac{(b_i - b_{m_I} - (b_j - b_{m_I}))^2}{I} - \sum_{i=1}^I \frac{(b_i - b_{m_I})^2}{I} \right] + \\
&+ \alpha_I E \left[ \frac{1}{2(d_j + 2)} - \frac{1}{2(d_{m_I} + 2)} \right] \\
&= \alpha_I E \left[ \sum_{i=1}^I \frac{(b_i - b_{m_I})^2 + (b_j - b_{m_I})^2 - 2(b_i - b_{m_I})(b_j - b_{m_I})}{I} - \sum_{i=1}^I \frac{(b_i - b_{m_I})^2}{I} \right] \\
&+ \alpha_I E \left[ \frac{1}{2(d_j + 2)} - \frac{1}{2(d_{m_I} + 2)} \right]
\end{aligned}$$

Since, by Lemma 4,  $\lim_{I \rightarrow \infty} \Pr(d_{m_I} \leq d) = 0$  for all  $d > 0$ , it follows that  $\lim_{I \rightarrow \infty} E \left[ \frac{1}{2(d_{m_I} + 2)} \right] = 0$ .

Further,  $\lim_{I \rightarrow \infty} m_I = E[b_i] = \lim_{I \rightarrow \infty} E \left[ \sum_{i=1}^I \frac{b_i}{I} \right]$ . Using these facts we have that

$$\lim_{I \rightarrow \infty} W_I^{m_I} - W_I^{\alpha \varepsilon} \geq \lim_{I \rightarrow \infty} \alpha \sum_{i=1}^I \frac{(b_j - b_{m_I})^2}{I} \geq \alpha \delta^2 > 0.$$

This result implies the as  $I$  approaches infinity, all decisions are optimally concentrated to politicians sufficiently close to the most moderate agent  $m_I$ . This concludes the proof of proposition 4.  $\blacksquare$

**Proof of Proposition 6.** First note that if there is  $i > 1$  such that  $b_i - b_{i-1} \leq 1/6$ , then  $i$  is informed of  $i - 1$ 's message or viceversa. For generic assignments of  $\mathbf{b}$ , it cannot be the case that  $\left| \sum_{j=1}^I \gamma_j b_j - b_i \right| = \left| \sum_{j=1}^I \gamma_j b_j - b_{i-1} \right|$ . Supposing without loss of generality that  $\left| \sum_{j=1}^I \gamma_j b_j - b_i \right| < \left| \sum_{j=1}^I \gamma_j b_j - b_{i-1} \right|$ , it is therefore welfare superior to assign  $a(i - 1) = i$  rather than  $a(i - 1) = i - 1$ . So suppose that  $b_i - b_{i-1} > 1/6$  for all  $i$ , so that for all  $j \neq i$ ,  $d_{i,j}(\mathbf{m}) = 0$  in any equilibrium  $(\mathbf{m}, \mathbf{y})$ . Hence, assigning  $a(1) = \lfloor (I + 1)/2 \rfloor \equiv m^*$  yields higher welfare than  $a(1) = 1$  if and only if:

$$\sum_{i=1}^I \frac{(b_i - b_1)^2}{I} - \sum_{i=1}^I \frac{(b_i - b_{m^*})^2}{I} > \frac{1}{36}.$$

The left-hand side can be rewritten as:

$$D(\Delta) = \sum_{i=2}^I \left[ (i-1) \frac{1}{6} + \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) \right]^2 - \sum_{i=m+1}^I \left[ (i-m) \frac{1}{6} + \sum_{j=m+1}^i \left( \Delta_j - \frac{1}{6} \right) \right]^2 - \sum_{i=1}^{m-1} \left[ (m-i) \frac{1}{6} + \sum_{j=i+1}^m \left( \Delta_j - \frac{1}{6} \right) \right]^2,$$

where  $\Delta_2 = b_2 - b_1, \dots, \Delta_I = b_I - b_{I-1}$ .

We now show that  $D(\Delta)$  increases in all its terms  $\Delta_k$ .

When  $k > m$ , we obtain:

$$\frac{\partial}{\partial \Delta_k} D(\Delta) = 2 \sum_{i=k}^I \left[ (i-1) \frac{1}{6} + \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) \right] - 2 \sum_{i=k}^I \left[ (i-m) \frac{1}{6} + \sum_{j=m+1}^i \left( \Delta_j - \frac{1}{6} \right) \right]$$

which is clearly positive because  $m > 1$  and  $m+1 \geq 2$ .

When  $k = m$ , we have

$$\frac{\partial}{\partial \Delta_k} D(\Delta) = 2 \sum_{i=k}^I \left[ (i-1) \frac{1}{6} + \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) \right] > 0$$

Suppose finally that  $k < m$ ,

$$\begin{aligned} \frac{\partial}{\partial \Delta_k} D(\Delta) &= 2 \sum_{i=k}^I \left[ (i-1) \frac{1}{6} + \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) \right] - 2 \sum_{i=1}^{k-1} \left[ (m-i) \frac{1}{6} + \sum_{j=i+1}^m \left( \Delta_j - \frac{1}{6} \right) \right] \\ (11) \quad &= 2 \sum_{i=k}^I (i-1) \frac{1}{6} - 2 \sum_{i=1}^{k-1} (m-i) \frac{1}{6} \end{aligned}$$

$$(12) \quad + 2 \sum_{i=k}^I \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) - 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^m \left( \Delta_j - \frac{1}{6} \right).$$

Because,  $k < m$ , evidently,

$$\begin{aligned} 2 \sum_{i=k}^I (i-1) \frac{1}{6} &> 2 \sum_{i=m+1}^I (i-1) \frac{1}{6} > 2 \sum_{i=m+1}^I (i-m) \frac{1}{6}, \\ \text{and } 2 \sum_{i=1}^{k-1} (m-i) \frac{1}{6} &< 2 \sum_{i=1}^{m-1} (m-i) \frac{1}{6}. \end{aligned}$$

and hence expression (11) is strictly positive. Further

$$\begin{aligned} 2 \sum_{i=k}^I \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) &> 2 \sum_{i=k}^m \sum_{j=2}^i \left( \Delta_j - \frac{1}{6} \right) \\ &= 2 \sum_{i=2}^k \sum_{j=i}^m \left( \Delta_j - \frac{1}{6} \right) = 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^m \left( \Delta_j - \frac{1}{6} \right) \end{aligned}$$

and hence expression (12) is strictly positive, concluding that  $\partial D(\Delta) / \partial \Delta_k$  is strictly positive.

Hence, we may take  $\Delta = \mathbf{1}/6$ , so that

$$D(\mathbf{1}/6) = \sum_{i=2}^I \left[ (i-1) \frac{1}{6} \right]^2 - \sum_{i=m+1}^I \left[ (i-m) \frac{1}{6} \right]^2 - \sum_{i=1}^{m-1} \left[ (m-i) \frac{1}{6} \right]^2,$$

Noting that for  $I$  odd,

$$D(\mathbf{1}/6) = \sum_{i=2}^I \left[ (i-1) \frac{1}{6} \right]^2 - 2 \sum_{i=m+1}^I \left[ (i-m) \frac{1}{6} \right]^2 = \frac{1}{4} I(I-1)^2 \frac{1}{36} \geq \frac{1}{4} \cdot 3 \cdot 4 \frac{1}{36} > \frac{1}{36}.$$

and for  $I$  even,

$$D(\mathbf{1}/6) = \sum_{i=2}^I (i-1)^2 - \sum_{i=I/2+1}^I (i-I/2)^2 - \sum_{i=1}^{I/2-1} (I/2-i)^2 = \frac{1}{4} I^2 (I-2) \frac{1}{36} \geq \frac{1}{4} \cdot 16 \cdot 2 \frac{1}{36} > \frac{1}{36},$$

we conclude that  $a(1) = \lfloor (I+1)/2 \rfloor \equiv m^*$  yields higher welfare than  $a(1) = 1$ .  $\blacksquare$

**Proof of Proposition 7.** Let  $\{f_1, f_2, \dots, f_n, \dots\}$  be an infinite sequence of density functions with the property that the mean is constant along the sequence, i.e.,  $\mu_l = m^*$  for all  $l = 1, \dots, \infty$ . Let  $\sigma_l^2$  be the variance associated to density  $f_l$  in the sequence. Similarly, let  $\Pr_l(a(k) = m^*)$  the concentration of policies when the density is  $f_l$ .

We now show that if along the sequence  $\{f_1, f_2, \dots, f_n, \dots\}$  the variance decreases,  $\sigma_n^2 > \sigma_{n+1}^2$ , and as  $n$  goes to infinity  $\sigma_n^2$  converges to zero, then, for every  $\epsilon > 0$ , there exists a  $\bar{n}$  such that  $\Pr_n(a(k) = m^*) > 1 - \epsilon$  for all  $n > \bar{n}$ .

Note that

$$\int_{\mu+1/6}^{\infty} (x-\mu)^2 f_n(x) dx \geq \frac{1}{36} \int_{\mu+1/6}^{\infty} f_n(x) dx.$$

Furthermore, by assumption, for every  $\epsilon' > 0$ , there exists a  $n_{\epsilon'}$  such that:

$$\sigma_n^2 = \int_{-\infty}^{\mu-1/6} (x-\mu)^2 f_n(x) dx + \int_{\mu-1/6}^{\mu+1/6} (x-\mu)^2 f_n(x) dx + \int_{\mu+1/6}^{\infty} (x-\mu)^2 f_n(x) dx < \epsilon'$$

for all  $n > n_{\epsilon'}$ . These two facts together imply that

$$\frac{1}{36} \int_{\mu+1/6}^{\infty} f_n(x) dx < \epsilon'$$

or

$$\int_{\mu+1/6}^{\infty} f_n(x) dx < 36\epsilon'$$

Similarly, one obtains that

$$\int_{-\infty}^{\mu-1/6} f_n(x) dx < 36\epsilon'$$

Define  $\epsilon = 72\epsilon'$  then we have that  $\Pr_n(a(k) = m^*) > 1 - \epsilon$ .

The second part of the proposition is proved as follows. Note that as  $f_n$  is single picked

$$\int_{\mu-1/3}^{\mu+1/3} f_n(x) dx < 2/3 \hat{f}_n,$$

where  $\hat{f}_n$  denote the max of  $f_n$ . It is then clear that if, as  $n$  goes to infinity,  $\hat{f}_n$  goes to zero, then the above integral tends to zero and therefore the complement tends to 1. ■

### **Proof that Skew Normal Distributions are Covered by Proposition 7.**

We want to show that if  $f_n$  is the density of a skewed normal, then its maximum tends to zero along the sequence defined in the proof of Proposition 7.

The density of the skewed normal distribution is

$$f_n(x) = \frac{1}{\omega_n \pi} e^{-\frac{x^2}{2\omega_n^2}} \int_{-\infty}^{\frac{\alpha_n x}{\omega_n}} e^{-\frac{t^2}{2}} dt$$

Let  $\delta_n = \frac{\alpha_n}{\sqrt{1+\alpha_n^2}}$ , then the mean is

$$\mu_n = \omega_n \delta_n \sqrt{\frac{2}{\pi}}$$

and the variance is

$$\sigma_n^2 = \omega_n^2 \left( 1 - \frac{2\delta_n^2}{\pi} \right)$$

Since the mean is constant along the infinite sequence  $\{f_1, f_2, \dots, f_n, \dots\}$ , i.e.,  $\mu_n = m^*$  for all  $n$ , we have that  $\omega_n$  and  $\delta_n$  move according to

$$\mu = \omega_n \delta_n \sqrt{\frac{2}{\pi}}.$$

By assumption as  $n$  goes to infinity the variance goes to infinity, i.e.,  $\lim_{n \rightarrow \infty} \sigma_n^2 = \infty$ . Since  $\delta_n \in [0, 1)$ , this implies that  $\lim_{n \rightarrow \infty} \omega_n = \infty$ , and since the mean is constant we also have that  $\lim_{n \rightarrow \infty} \delta_n = 0$  or equivalently  $\lim_{n \rightarrow \infty} \alpha_n = 0$ . Note also that

$$\lim_{n \rightarrow \infty} \frac{\alpha_n}{\omega_n} = \lim_{n \rightarrow \infty} \frac{\alpha_n}{\frac{\mu\sqrt{\pi/2}}{\delta_n}} = 0.$$

Next, let  $x_n$  be the maximum of the density  $f_n$ . As  $f_n$  is singled picked,  $x_n$  is the solution to  $df_n/dx_n = 0$ , or equivalently:

$$-\frac{x_n}{\omega} \int_{-\infty}^{\frac{\alpha x}{\omega}} e^{-\frac{t^2}{2}} dt + \alpha e^{-\frac{\alpha^2 x_n^2}{2\omega^2}} = 0$$

Two observations follow. The first observation is that when  $n$  goes to infinity,  $\alpha x_n/\omega_n$  is bounded. Suppose the contrary, then the second term in the expression above goes to zero, whereas the first term goes to  $-\infty$  because  $-x_n/\omega_n < -\alpha_n x_n/\omega_n$  which goes to  $-\infty$  and all multiplies a strictly positive number. Second, since as  $n$  goes to infinity  $\alpha x_n/\omega_n$  is bounded and  $\alpha_n$  goes to zero, we have that

$$-\frac{x_n}{\omega} \int_{-\infty}^0 e^{-\frac{t^2}{2}} dt = 0$$

which implies that  $x_n$  goes to 0 as  $n$  goes to infinity. Using these observations we conclude that

$$\lim_{n \rightarrow \infty} f_n(x_n) = \lim_{n \rightarrow \infty} \frac{1}{\omega_n \pi} e^{-\frac{x_n^2}{2\omega_n^2}} \int_{-\infty}^{\frac{\alpha_n x_n}{\omega_n}} e^{-\frac{t^2}{2}} dt = 0$$

as  $1/\omega_n$  tends to 0 and the rest is bounded. ■

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